

thm_2Ewords_2Esw2sw_id
(TMUdyb6DVqqAsEcnA9qoy8tfHfDYinbqvJu)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 7 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 8 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Definition 9 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 10 We define c_Enum_ESUC to be $\lambda V0m \in ty_Enum_Enum.(ap\ c_Enum_EABS_num$
Let $c_Earithmic_E_2B : \iota$ be given. Assume the following.

$$c_Earithmic_E_2B \in ((ty_Enum_Enum^{ty_Enum_Enum})^{ty_Enum_Enum}) \quad (6)$$

Definition 11 We define $c_Earithmic_EBIT1$ to be $\lambda V0n \in ty_Enum_Enum.(ap\ (ap\ c_Earithmic_E_2B$

Definition 12 We define $c_Earithmic_ENUMERAL$ to be $\lambda V0x \in ty_Enum_Enum.V0x$.

Let $c_Earithmic_E_2D : \iota$ be given. Assume the following.

$$c_Earithmic_E_2D \in ((ty_Enum_Enum^{ty_Enum_Enum})^{ty_Enum_Enum}) \quad (7)$$

Let $ty_Ebool_Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_Ebool_Eitself\ A0) \quad (8)$$

Let $c_Ebool_Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_Ebool_Ethe_value\ A_27a \in (ty_Ebool_Eitself\ A_27a) \quad (9)$$

Let $c_Efcp_Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_Efcp_Edimindex\ A_27a \in (ty_Enum_Enum^{(ty_Ebool_Eitself\ A_27a)}) \quad (10)$$

Let $ty_Efcp_Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_Efcp_Efinite_image\ A0) \quad (11)$$

Definition 13 We define $c_Emin_E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A \wedge P\ x))$
of type $\iota \Rightarrow \iota$.

Definition 14 We define $c_Ebool_E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_Emin_E_40\ A_27a\ P))$

Definition 15 We define $c_Eprim_rec_E_3C$ to be $\lambda V0m \in ty_Enum_Enum.\lambda V1n \in ty_Enum_Enum$

Definition 16 We define $c_Ebool_E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ c_Ebool_E_2F_5C\ A_27a\ P))$

Definition 17 We define $c_Efcp_Efinite_index$ to be $\lambda A_27a : \iota.(ap\ (c_Emin_E_40\ (A_27a^{ty_Enum_Enum}))$

Let $ty_Efcp_Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_Efcp_Ecart\ A0\ A1) \quad (12)$$

Let $c_Efcp_Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_Efcp_Edest_cart\ A_27a\ A_27b \in ((A_27a^{(ty_Efcp_Efinite_image\ A_27b)})^{(ty_Efcp_Ecart\ A_27a\ A_27b)}) \quad (13)$$

Definition 18 We define $c_2Efc_2Efc_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efc_2Ecart\ 2\ A_27a).$

Definition 19 We define $c_2Ewords_2Eword_msb$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efc_2Ecart\ 2\ A_27a).$

Definition 20 We define $c_2Ebool_2E5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in 2.$

Definition 21 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.$

Definition 22 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (14)$$

Definition 23 We define c_2Ebit_2ESBIT to be $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ (c_2Ebo$

Let $c_2Esum_num_2ESUM : \iota$ be given. Assume the following.

$$c_2Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (15)$$

Definition 24 We define $c_2Ewords_2Ew2n$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efc_2Ecart\ 2\ A_27a).(ap\ (ap\ c$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (16)$$

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (17)$$

Definition 25 We define $c_2Ebit_2EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Definition 26 We define $c_2Ebit_2EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Definition 27 We define c_2Ebit_2EBITS to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.\lambda V$

Definition 28 We define c_2Ebit_2EBIT to be $\lambda V0b \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap$

Definition 29 We define c_2Ebool_2ELET to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in (A_27b^{A_27a}).(\lambda V1x \in A_27$

Definition 30 We define $c_2Ebit_2ESIGN_EXTEND$ to be $\lambda V0l \in ty_2Enum_2Enum.\lambda V1h \in ty_2Enum_2$

Definition 31 We define c_2Efc_2EFCP to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap$

Definition 32 We define $c_2Ewords_2En2w$ to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.(ap\ (c_2Efc_2EFC$

Definition 33 We define $c_2Ewords_2Esw2sw$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0w \in (ty_2Efc_2Ecart\ 2\ A_2$

Assume the following.

$$True \quad (18)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2)))))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p \ V0t))) \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p \ V0t)) \Leftrightarrow (p \ V0t))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow False) \Leftrightarrow (\neg (p \ V0t)))))) \quad (23)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (27)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (28)$$

Assume the following.

$$2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))) \Rightarrow 2.(((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))) \quad (29)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\ & (\forall V2x \in A_{.27a}.(\forall V3x_{.27} \in A_{.27a}.(\forall V4y \in A_{.27a}. \\ & (\forall V5y_{.27} \in A_{.27a}.(((p V0P) \Leftrightarrow (p V1Q)) \wedge ((p V1Q) \Rightarrow (V2x = V3x_{.27})) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y_{.27})))) \Rightarrow ((ap (ap (ap (c_{.2Ebool_{.2ECOND}} A_{.27a}) \\ & V0P) V2x) V4y) = (ap (ap (ap (c_{.2Ebool_{.2ECOND}} A_{.27a}) V1Q) V3x_{.27}) \\ & V5y_{.27})))))))))) \quad (30) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty A_{.27a} \Rightarrow ((\forall V0t1 \in A_{.27a}.(\forall V1t2 \in \\ & A_{.27a}.((ap (ap (ap (c_{.2Ebool_{.2ECOND}} A_{.27a}) c_{.2Ebool_{.2ET}} V0t1) \\ & V1t2) = V0t1))) \wedge (\forall V2t1 \in A_{.27a}.(\forall V3t2 \in A_{.27a}.((ap \\ & (ap (ap (c_{.2Ebool_{.2ECOND}} A_{.27a}) c_{.2Ebool_{.2EF}} V2t1) V3t2) = V3t2)))))) \quad (31) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\\ & \forall V0x \in (ty_{.2Efc}_{.2Ecart} A_{.27a} A_{.27b}).(\forall V1y \in (ty_{.2Efc}_{.2Ecart} \\ & A_{.27a} A_{.27b}).((V0x = V1y) \Leftrightarrow (\forall V2i \in ty_{.2Enum_{.2Enum}}.((p (ap \\ & (ap c_{.2Eprim_{.rec_{.2E_{.3C}}} V2i) (ap (c_{.2Efc}_{.2Edimindex} A_{.27b}) (\\ & c_{.2Ebool_{.2Ethe_{.value}} A_{.27b})))) \Rightarrow ((ap (ap (c_{.2Efc}_{.2Efc}_{.index} \\ & A_{.27a} A_{.27b}) V0x) V2i) = (ap (ap (c_{.2Efc}_{.2Efc}_{.index} A_{.27a} A_{.27b}) \\ & V1y) V2i)))))) \quad (32) \end{aligned}$$

Assume the following.

$$(\forall V0n \in ty_{.2Enum_{.2Enum}}.(\neg(p (ap (ap c_{.2Eprim_{.rec_{.2E_{.3C}}} V0n) V0n)))) \quad (33)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a). (\forall V1i \in ty_2Enum_2Enum. \\
& \quad ((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V1i)\ (ap\ (c_2EfcP_2Edimindex\ A_27b) \\
& \quad (c_2Ebool_2Ethe_value\ A_27b)))) \Rightarrow ((p\ (ap\ (ap\ (c_2EfcP_2EfcP_index \\
& \quad 2\ A_27b)\ (ap\ (c_2Ewords_2Esw2sw\ A_27a\ A_27b)\ V0w))\ V1i)) \Leftrightarrow (p\ (ap \\
& \quad (ap\ (ap\ (c_2Ebool_2ECOND\ 2)\ (ap\ (ap\ c_2Ebool_2E_5C_2F\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& \quad V1i)\ (ap\ (c_2EfcP_2Edimindex\ A_27a)\ (c_2Ebool_2Ethe_value\ A_27a)))) \\
& \quad (ap\ (ap\ c_2Eprim_rec_2E_3C\ (ap\ (c_2EfcP_2Edimindex\ A_27b)\ (c_2Ebool_2Ethe_value \\
& \quad A_27b))))\ (ap\ (c_2EfcP_2Edimindex\ A_27a)\ (c_2Ebool_2Ethe_value \\
& \quad A_27a))))\ (ap\ (ap\ (c_2EfcP_2EfcP_index\ 2\ A_27a)\ V0w)\ V1i))\ (\\
& \quad ap\ (c_2Ewords_2Eword_msb\ A_27a)\ V0w))))))
\end{aligned} \tag{34}$$

Theorem 1

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a). ((ap\ (c_2Ewords_2Esw2sw\ A_27a\ A_27a)\ V0w) = V0w))$$