

thm\_2Ewords\_2Ew2w\_eq\_n2w (TMQFT-tqxcp4tqXQH2vSGMuKU8UhHfQiJb8y)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})) (\lambda V1t \in 2.V1t)) P)))$

**Definition 4** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2. (ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2EF))$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2. (ap (c\_2Ebool\_2E\_7E V1t2) c\_2Ebool\_2EF)) V0t1))))$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (2)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (3)$$

Let  $c\_2Enum\_2EAbs\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EAbs\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (4)$$

**Definition 8** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap c\_2Enum\_2EAbs\_num m)$

**Definition 9** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ .

**Definition 10** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40\ A\_27a)\ V0P)))$

**Definition 11** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum. (c\_2Eprim\_rec\ V0m\ V1n)$

**Definition 12** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(V1t1\ t2)))$

**Definition 13** We define  $c\_2Earithmetic\_2EMIN$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum. (c\_2Earithmetic\_2EMIN\ V0m\ V1n)$

Let  $ty\_2Efcp\_2Efinite\_image : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efcp\_2Efinite\_image\ A0) \quad (5)$$

Let  $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ebool\_2Eitself\ A0) \quad (6)$$

Let  $c\_2Ebool\_2Ethethe\_value : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2Ethethe\_value\ A\_27a \in (ty\_2Ebool\_2Eitself\ A\_27a) \quad (7)$$

Let  $c\_2Efcp\_2Edimindex : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Efcp\_2Edimindex\ A\_27a \in (ty\_2Enum\_2Enum)^{(ty\_2Ebool\_2Eitself\ A\_27a)} \quad (8)$$

**Definition 14** We define  $c\_2Ebool\_2E\_3F\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ c\_2Ebool\_2E\_2F\_5C\ A\_27a)\ V0P)))$

**Definition 15** We define  $c\_2Efcp\_2Efinite\_index$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (c\_2Emin\_2E\_40\ (A\_27a^{ty\_2Enum\_2Enum}))\ V0P)))$

Let  $ty\_2Efcp\_2Ecarts : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efcp\_2Ecarts\ A0\ A1) \quad (9)$$

Let  $c\_2Efcp\_2Edests\_cart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efcp\_2Edests\_cart\ A\_27a\ A\_27b \in ((A\_27a^{(ty\_2Efcp\_2Efinite\_image\ A\_27b)})^{(ty\_2Efcp\_2Ecarts\ A\_27a\ A\_27b)}) \quad (10)$$

**Definition 16** We define  $c\_2Efcp\_2Efcp\_index$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in (ty\_2Efcp\_2Ecarts\ A\_27a\ A\_27b). (c\_2Efcp\_2Efcp\ V0x)$

Let  $c\_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (11)$$

Let  $c\_2Earithmetic\_2EODD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (12)$$

**Definition 17** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum. (c\_2Earithmetic\_2E\_3E\ V0m\ V1n)$

**Definition 18** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap(c\_2Ebool\_2E\_21 2))(\lambda V2t \in$

**Definition 19** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (13)$$

**Definition 20** We define  $c\_2Enum\_2E0$  to be  $(ap(c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP))$

**Definition 21** We define  $c\_2Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap(ap(ap(c\_2Ebool\_2E$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (14)$$

**Definition 22** We define  $c\_2Enumeral\_2EiiSUC$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap(c\_2Enum\_2ESUC)(ap$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (15)$$

**Definition 23** We define  $c\_2Enumeral\_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

**Definition 24** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap(ap(c\_2Earithmetic$

Let  $c\_2Earithmetic\_2EEEXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (16)$$

**Definition 25** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0.$

**Definition 26** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap(ap(c\_2Earithmetic$

**Definition 27** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let  $c\_2Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (17)$$

**Definition 28** We define  $c\_2Ebit\_2EDIV\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (18)$$

Let  $c\_2Earithmetic\_2EMOD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (19)$$

**Definition 29** We define  $c\_2Ebit\_2EMOD\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 30** We define  $c\_2Ebit\_2EBITS$  to be  $\lambda V0h \in ty\_2Enum\_2Enum. \lambda V1l \in ty\_2Enum\_2Enum. \lambda V2m \in ty\_2Enum\_2Enum.$

**Definition 31** We define  $c\_2Ebit\_2EBIT$  to be  $\lambda V0b \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum. (ap$

**Definition 32** We define  $c\_2Efcp\_2EFCP$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}). (ap$

Let  $c\_2Ewords\_2Edimword : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Ewords\_2Edimword\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (20)$$

**Definition 33** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 34** We define  $c\_2Ewords\_2En2w$  to be  $\lambda A\_27a : \iota. \lambda V0n \in ty\_2Enum\_2Enum. (ap$

**Definition 35** We define  $c\_2Ebit\_2ESBIT$  to be  $\lambda V0b \in 2. \lambda V1n \in ty\_2Enum\_2Enum. (ap$

Let  $c\_2Esum\_num\_2ESUM : \iota$  be given. Assume the following.

$$c\_2Esum\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})^{ty\_2Enum\_2Enum}) \quad (21)$$

**Definition 36** We define  $c\_2Ewords\_2Ew2n$  to be  $\lambda A\_27a : \iota. \lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a). (ap$

**Definition 37** We define  $c\_2Ewords\_2Ew2w$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).$

Assume the following.

$$((ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)) = \\ (ap\ c\_2Enum\_2ESUC\ c\_2Enum\_2E0)) \quad (22)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. ((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m) \\ c\_2Enum\_2E0) = V0m)) \quad (23)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (\\ ((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Enum\_2E0)\ V0m) = V0m) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Enum\_2E0)\ V0m) = V0m) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Enum\_2E0)\ V1n) = (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Enum\_2E0)\ V1n))) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Enum\_2E0)\ V1n) = (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Enum\_2E0)\ V1n))))))) \quad (24)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (\\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V0m)))) \quad (25)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (\forall V2p \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2B V0m) \\
 & (ap (ap c\_2Earithmetic\_2E\_2B V1n) V2p)) = (ap (ap c\_2Earithmetic\_2E\_2B \\
 & (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) V2p)))))) \\
 \end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (\forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
 & (ap c\_2Enum\_2ESUC V0m)) V1n)))))) \\
 \end{aligned} \tag{27}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
 c\_2Enum\_2E0) V0n))) \tag{28}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (\forall V2p \in ty\_2Enum\_2Enum. ((\neg(p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
 & V1n) V0m)))))) \\
 \end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (\forall V2p \in ty\_2Enum\_2Enum. ((((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V0m) = c\_2Enum\_2E0) \wedge \\
 & (((ap (ap c\_2Earithmetic\_2E\_2A V0m) c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge \\
 & (((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL \\
 & (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0m) = V0m) \wedge \\
 & (((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Earithmetic\_2ENUMERAL \\
 & (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = V0m) \wedge \\
 & ((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap \\
 & (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)) \\
 & V1n)) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Enum\_2ESUC V1n)) = \\
 & (ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap (ap c\_2Earithmetic\_2E\_2A \\
 & V0m) V1n)))))))))) \\
 \end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (\forall V2p \in ty\_2Enum\_2Enum. (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
 & V0m) V1n)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p))) \Rightarrow (p ( \\
 & ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V2p)))))) \\
 \end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
 & (V0m = V1n) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)) \wedge (p \\
 & \quad ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V0m)))))) \\
 \end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
 & \forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
 & \quad ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) (ap (ap c\_2Earithmetic\_2E\_2B \\
 & \quad V0m) V2p))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p)))))) \\
 \end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
 & (\neg(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
 & \quad (ap c\_2Enum\_2ESUC V1n)) V0m)))))) \\
 \end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
 & (\neg(V0m = V1n)) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC \\
 & \quad V0m)) V1n)) \vee (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC \\
 & \quad V1n)) V0m)))))) \\
 \end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty\_2Enum\_2Enum. ((ap c\_2Enum\_2ESUC V0n) = (ap (ap \\
 & \quad c\_2Earithmetic\_2E\_2B (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
 & \quad c\_2Earithmetic\_2EZERO)) V0n))) \\
 \end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
 & \forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) \\
 & \quad (ap (ap c\_2Earithmetic\_2E\_2D V1n) V2p))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
 & \quad (ap (ap c\_2Earithmetic\_2E\_2B V0m) V2p)) V1n)))))) \\
 \end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
 & \forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
 & \quad ap (ap c\_2Earithmetic\_2E\_2D V0m) V1n)) V2p)) \Leftrightarrow ((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
 & \quad V0m) (ap (ap c\_2Earithmetic\_2E\_2B V1n) V2p))) \wedge (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
 & \quad c\_2Enum\_2E0) V2p)))))) \\
 \end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1k \in ty\_2Enum\_2Enum. ( \\
 & (p (ap (ap c\_2Eprim\_rec\_2E\_3C V1k) V0n)) \Rightarrow ((ap (ap c\_2Earithmetic\_2EMOD \\
 & V1k) V0n) = V1k)))) \\
 \end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0P \in (2^{ty\_2Enum\_2Enum}). (\forall V1a \in ty\_2Enum\_2Enum. \\
 & (\forall V2b \in ty\_2Enum\_2Enum. ((p (ap V0P (ap (ap c\_2Earithmetic\_2E\_2D \\
 & V1a) V2b))) \Leftrightarrow (\forall V3d \in ty\_2Enum\_2Enum. ((V2b = (ap (ap c\_2Earithmetic\_2E\_2B \\
 & V1a) V3d)) \Rightarrow (p (ap V0P c\_2Enum\_2E0))) \wedge ((V1a = (ap (ap c\_2Earithmetic\_2E\_2B \\
 & V2b) V3d)) \Rightarrow (p (ap V0P V3d))))))) \\
 \end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0a \in ty\_2Enum\_2Enum. (\forall V1b \in ty\_2Enum\_2Enum. ( \\
 & (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0a) V1b)) \Rightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
 & (ap (ap c\_2Earithmetic\_2EEEXP (ap c\_2Earithmetic\_2ENUMERAL (ap \\
 & c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) V0a)) (ap (ap \\
 & c\_2Earithmetic\_2EEEXP (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 \\
 & c\_2Earithmetic\_2EZERO))) V1b))))))) \\
 \end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0h \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
 & (ap (ap (ap c\_2Ebit\_2EBITS V0h) c\_2Enum\_2E0) V1n) = (ap (ap c\_2Earithmetic\_2EMOD \\
 & V1n) (ap (ap c\_2Earithmetic\_2EEEXP (ap c\_2Earithmetic\_2ENUMERAL \\
 & (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) (ap c\_2Enum\_2ESUC \\
 & V0h))))))) \\
 \end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0h \in ty\_2Enum\_2Enum. (\forall V1a \in ty\_2Enum\_2Enum. ( \\
 & (p (ap (ap c\_2Eprim\_rec\_2E\_3C V1a) (ap (ap c\_2Earithmetic\_2EEEXP \\
 & (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) \\
 & (ap c\_2Enum\_2ESUC V0h)))) \Rightarrow ((ap (ap (ap c\_2Ebit\_2EBITS V0h) c\_2Enum\_2E0) \\
 & V1a) = V1a)))) \\
 \end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0h1 \in ty\_2Enum\_2Enum. (\forall V1l1 \in ty\_2Enum\_2Enum. \\
 & (\forall V2h2 \in ty\_2Enum\_2Enum. (\forall V3l2 \in ty\_2Enum\_2Enum. \\
 & (\forall V4n \in ty\_2Enum\_2Enum. ((ap (ap c\_2Ebit\_2EBITS V2h2) \\
 & V3l2) (ap (ap (ap c\_2Ebit\_2EBITS V0h1) V1l1) V4n)) = (ap (ap (ap c\_2Ebit\_2EBITS \\
 & (ap (ap c\_2Earithmetic\_2EMIN V0h1) (ap (ap c\_2Earithmetic\_2E\_2B \\
 & V2h2) V1l1))) (ap (ap c\_2Earithmetic\_2E\_2B V3l2) V1l1)) V4n))))))) \\
 \end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0h \in ty\_2Enum\_2Enum. (\forall V1l \in ty\_2Enum\_2Enum. \\
 & \quad \forall V2a \in ty\_2Enum\_2Enum. (\forall V3b \in ty\_2Enum\_2Enum. (( \\
 & \quad \quad \forall V4x \in ty\_2Enum\_2Enum. (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
 & \quad \quad \quad V1l) V4x)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V4x) V0h))) \Rightarrow ((p \\
 & \quad \quad \quad (ap (ap c\_2Ebit\_2EBIT V4x) V2a)) \Leftrightarrow (p (ap (ap c\_2Ebit\_2EBIT V4x) V3b)))))) \Leftrightarrow \\
 & \quad \quad \quad ((ap (ap (ap c\_2Ebit\_2EBITS V0h) V1l) V2a) = (ap (ap (ap c\_2Ebit\_2EBITS \\
 & \quad \quad \quad V0h) V1l) V3b)))))))
 \end{aligned} \tag{45}$$

Assume the following.

$$True \tag{46}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \tag{47}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \tag{48}$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \tag{49}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a. (p V0t)) \Leftrightarrow (p V0t))) \tag{50}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \wedge \\
 & \quad ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \tag{51}$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \tag{52}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \tag{53}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\
 & \quad (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\
 & \quad (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t))))))
 \end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t))))))) \end{aligned} \quad (55)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow true) \Leftrightarrow \\ & true) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow true) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow true) \wedge (( \\ & (p V0t) \Rightarrow false) \Leftrightarrow (\neg(p V0t))))))) \end{aligned} \quad (56)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t)) \Leftrightarrow (p V0t))) \wedge (((\neg true) \Leftrightarrow false) \wedge \\ ((\neg false) \Leftrightarrow true))) \quad (57))$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow \\ true)) \quad (58)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \\ A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (59)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow true) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow false) \Leftrightarrow (\neg \\ & (p V0t))))))) \end{aligned} \quad (60)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in \\ & A\_27a.(((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2ET) V0t1) \\ & V1t2) = V0t1) \wedge ((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2EF) \\ & V0t1) V1t2) = V1t2)))))) \end{aligned} \quad (61)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg(\forall V1x \in \\ & A\_27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A\_27a.(\neg(p (ap V0P V2x))))))) \end{aligned} \quad (62)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in \\ & 2.(((\forall V2x \in A\_27a.(p (ap V0P V2x))) \wedge (p V1Q)) \Leftrightarrow (\forall V3x \in \\ & A\_27a.((p (ap V0P V3x)) \wedge (p V1Q))))) \end{aligned} \quad (63)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in \\ 2.(((\exists V2x \in A\_27a.(p (ap V0P V2x))) \vee (p V1Q)) \Leftrightarrow (\exists V3x \in \\ A\_27a.((p (ap V0P V3x)) \vee (p V1Q)))))) \end{aligned} \quad (64)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & (\forall V0P \in 2.(\forall V1Q \in ( \\ 2^{A\_27a}).((\exists V2x \in A\_27a.((p V0P) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((p \\ V0P) \wedge (\exists V3x \in A\_27a.(p (ap V1Q V3x))))))) \end{aligned} \quad (65)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & (\forall V0Q \in 2.(\forall V1P \in ( \\ 2^{A\_27a}).((\forall V2x \in A\_27a.((p (ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in \\ A\_27a.(p (ap V1P V3x)) \vee (p V0Q)))))) \end{aligned} \quad (66)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee ( \\ (p V1B) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (67)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee ( \\ (p V0A)))))) \quad (68)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg( \\ p V0A)) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B))))))) \quad (69)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee ( \\ (p V1B)))))) \quad (70)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow \text{False}) \Leftrightarrow ((p V0t) \Leftrightarrow \text{False}))) \quad (71)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ( \\ (p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (72)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p \\ V0t1) \wedge (p V1t2)) \vee ((\neg(p V0t1)) \wedge (\neg(p V1t2))))))) \quad (73)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in \\ & 2. (((((p V0x) \Leftrightarrow (p V1x\_27)) \wedge ((p V1x\_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y\_27)))) \Rightarrow \quad (74) \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x\_27) \Rightarrow (p V3y\_27))))))) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A\_27a. (\forall V3x\_27 \in A\_27a. (\forall V4y \in A\_27a. \\ & (\forall V5y\_27 \in A\_27a. (((((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y\_27)))) \Rightarrow ((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) \\ & V0P) V2x) V4y) = (ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) V1Q) V3x\_27) \\ & V5y\_27))))))))))) \quad (75) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow ( \\ & \forall V0P \in ((2^{A\_27b})^{A\_27a}). ((\forall V1x \in A\_27a. (\exists V2y \in \\ & A\_27b. (p (ap (ap V0P V1x) V2y))) \Leftrightarrow (\exists V3f \in (A\_27b)^{A\_27a}). ( \\ & \forall V4x \in A\_27a. (p (ap (ap V0P V4x) (ap V3f V4x))))))) \quad (76) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow ( \\ & \forall V0x \in (ty\_2Efcp\_2Ecart A\_27a A\_27b). (\forall V1y \in (ty\_2Efcp\_2Ecart \\ & A\_27a A\_27b). ((V0x = V1y) \Leftrightarrow (\forall V2i \in ty\_2Enum\_2Enum. ((p (ap \\ & (ap c\_2Eprim\_rec\_2E\_3C V2i) (ap (c\_2Efcp\_2Edimindex A\_27b) ( \\ & c\_2Ebool\_2Ethe\_value A\_27b)))) \Rightarrow ((ap (ap (c\_2Efcp\_2Efcp\_index \\ & A\_27a A\_27b) V0x) V2i) = (ap (ap (c\_2Efcp\_2Efcp\_index A\_27a A\_27b) \\ & V1y) V2i))))))) \quad (77) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow ( \\ & \forall V0g \in (A\_27a^{ty\_2Enum\_2Enum}). (\forall V1i \in ty\_2Enum\_2Enum. \\ & ((p (ap (ap c\_2Eprim\_rec\_2E\_3C V1i) (ap (c\_2Efcp\_2Edimindex A\_27b) \\ & (c\_2Ebool\_2Ethe\_value A\_27b)))) \Rightarrow ((ap (ap (c\_2Efcp\_2Efcp\_index \\ & A\_27a A\_27b) (ap (c\_2Efcp\_2EFCP A\_27a A\_27b) V0g)) V1i) = (ap V0g \\ & V1i)))))) \quad (78) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap \\
& (ap c\_2Earithmetic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in \\
ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& (ap c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Earithmetic\_2ENUMERAL \\
V3m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enum\_2EiZ (ap \\
& (ap c\_2Earithmetic\_2E\_2B V2n) V3m))))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum. \\
& ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge \\
& ((\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum. \\
& ((\forall V7m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
& ap c\_2Earithmetic\_2ENUMERAL V6n)) (ap c\_2Earithmetic\_2ENUMERAL \\
V7m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2A \\
V6n) V7m)))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum. \\
& ((ap (ap c\_2Earithmetic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in \\
ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& (ap c\_2Earithmetic\_2ENUMERAL V10n)) (ap c\_2Earithmetic\_2ENUMERAL \\
V11m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D \\
V10n) V11m)))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP \\
c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap \\
& (ap c\_2Earithmetic\_2EEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
(ap c\_2Earithmetic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in \\
ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP V14n) c\_2Enum\_2E0) = \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge \\
& ((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum. \\
& ((ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL V15n)) \\
(ap c\_2Earithmetic\_2ENUMERAL V16m)) = (ap c\_2Earithmetic\_2ENUMERAL \\
(ap (ap c\_2Earithmetic\_2EEXP V15n) V16m)))))) \wedge (((ap c\_2Enum\_2ESUC \\
c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
c\_2Earithmetic\_2EZERO)))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum. \\
& (ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL V17n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
(ap c\_2Enum\_2ESUC V17n)))) \wedge (((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = \\
c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE \\
(ap c\_2Earithmetic\_2ENUMERAL V18n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
(ap c\_2Eprim\_rec\_2EPRE V18n)))))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum. \\
& (((ap c\_2Earithmetic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmetic\_2EZERO))) \wedge \\
& ((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0 = (ap c\_2Earithmetic\_2ENUMERAL \\
V20n)) \Leftrightarrow (V20n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum. \\
& ((\forall V22m \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL \\
V21n) = (ap c\_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
V23n) c\_2Enum\_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
V24n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
V24n)))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmetic\_2ENUMERAL \\
V25n)) (ap c\_2Earithmetic\_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
V25n) V26m)))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E \\
c\_2Enum\_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
V28n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
V28n)))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
V29n)) (ap c\_2Earithmetic\_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
V30m) V29n)))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
c\_2Enum\_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL \\
V32n)))) \wedge ((\forall V33n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
c\_2Enum\_2E0) V33n)) \Leftrightarrow False)) \wedge ((\forall V34n \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL \\
V34n)) \Leftrightarrow False)))))))
\end{aligned}$$

Assume the following.

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D c\_2Earithmetic\_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) c\_2Earithmetic\_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& (ap c\_2Earithmetic\_2EBIT2 V0n)) c\_2Earithmetic\_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow (\neg(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V1m) V0n)))) \wedge ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))))))))))) \\
\end{aligned} \tag{81}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{82}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{83}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \\
\end{aligned} \tag{84}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \\
\end{aligned} \tag{85}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{86}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p \\
& V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))) \\
\end{aligned} \tag{87}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \\
\end{aligned} \tag{88}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge ((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (89)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((p V0p) \vee (\neg(p V2r))) \wedge (( \\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (90)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (91)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow ((ap (c\_2Ewords\_2Edimword A\_27a) \\ & (c\_2Ebool\_2Ethe\_value A\_27a)) = (ap (ap c\_2Earithmetic\_2EEEXP \\ & (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) \\ & (ap (c\_2Efcp\_2Edimindex A\_27a) (c\_2Ebool\_2Ethe\_value A\_27a)))))) \end{aligned} \quad (92)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) \\ & (ap (c\_2Efcp\_2Edimindex A\_27a) (c\_2Ebool\_2Ethe\_value A\_27a)))) \end{aligned} \quad (93)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\exists V0m \in ty\_2Enum\_2Enum. ( \\ & (ap (c\_2Efcp\_2Edimindex A\_27a) (c\_2Ebool\_2Ethe\_value A\_27a)) = (ap c\_2Enum\_2ESUC V0m))) \end{aligned} \quad (94)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum. ( \\ & (ap (ap c\_2Earithmetic\_2EMOD V0n) (ap (c\_2Ewords\_2Edimword A\_27a) \\ & (c\_2Ebool\_2Ethe\_value A\_27a))) = (ap (ap (ap c\_2Ebit\_2EBITS ( \\ & ap (ap c\_2Earithmetic\_2E\_2D (ap (c\_2Efcp\_2Edimindex A\_27a) (c\_2Ebool\_2Ethe\_value \\ & A\_27a))) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\ & c\_2Earithmetic\_2EZERO)))) c\_2Enum\_2E0) V0n))) \end{aligned} \quad (95)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}.nonempty\ A_{27a} \Rightarrow \forall A_{27b}.nonempty\ A_{27b} \Rightarrow ( \\ & (p (ap (ap c_2Earithmetic_2E_3C_3D (ap (c_2Efcp_2Edimindex A_{27a}) \\ & (c_2Ebool_2Ethe_value A_{27a}))) (ap (c_2Efcp_2Edimindex A_{27b}) \\ & (c_2Ebool_2Ethe_value A_{27b})))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\ & (ap (c_2Ewords_2Edimword A_{27a}) (c_2Ebool_2Ethe_value A_{27a}))) \\ & (ap (c_2Ewords_2Edimword A_{27b}) (c_2Ebool_2Ethe_value A_{27b})))))) \end{aligned} \quad (96)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}.nonempty\ A_{27a} \Rightarrow (\forall V0w \in (ty_2Efcp_2Ecarr \\ & 2\ A_{27a}).(\exists V1n \in ty_2Enum_2Enum.((V0w = (ap (c_2Ewords_2En2w \\ & A_{27a})\ V1n)) \wedge (p (ap (ap c_2Eprim_rec_2E_3C\ V1n) (ap (c_2Ewords_2Edimword \\ & A_{27a}) (c_2Ebool_2Ethe_value A_{27a}))))))) \end{aligned} \quad (97)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}.nonempty\ A_{27a} \Rightarrow \forall A_{27b}.nonempty\ A_{27b} \Rightarrow ( \\ & \forall V0n \in ty_2Enum_2Enum.((ap (c_2Ewords_2En2w\ A_{27a}\ A_{27b}) \\ & (ap (c_2Ewords_2En2w\ A_{27a})\ V0n)) = (ap (ap (ap (c_2Ebool_2ECOND \\ & (ty_2Efcp_2Ecarr\ 2\ A_{27b})) (ap (ap c_2Earithmetic_2E_3C_3D ( \\ & ap (c_2Efcp_2Edimindex\ A_{27b}) (c_2Ebool_2Ethe_value\ A_{27b}))) \\ & (ap (c_2Efcp_2Edimindex\ A_{27a}) (c_2Ebool_2Ethe_value\ A_{27a}))) \\ & (ap (c_2Ewords_2En2w\ A_{27b})\ V0n)) (ap (c_2Ewords_2En2w\ A_{27b}) \\ & (ap (ap (ap c_2Ebit_2EBITS (ap (ap c_2Earithmetic_2E_2D (ap (c_2Efcp_2Edimindex \\ & A_{27a}) (c_2Ebool_2Ethe_value A_{27a}))) (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))\ c_2Enum_2E0) \\ & V0n)))))) \end{aligned} \quad (98)$$

### Theorem 1

$$\begin{aligned} & \forall A_{27a}.nonempty\ A_{27a} \Rightarrow \forall A_{27b}.nonempty\ A_{27b} \Rightarrow ( \\ & \forall V0x \in (ty_2Efcp_2Ecarr\ 2\ A_{27a}).(\forall V1y \in ty_2Enum_2Enum. \\ & (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap (c_2Efcp_2Edimindex \\ & A_{27a}) (c_2Ebool_2Ethe_value A_{27a}))) (ap (c_2Efcp_2Edimindex \\ & A_{27b}) (c_2Ebool_2Ethe_value A_{27b})))) \wedge (p (ap (ap c_2Eprim_rec_2E_3C \\ & V1y) (ap (c_2Ewords_2Edimword A_{27a}) (c_2Ebool_2Ethe_value \\ & A_{27a})))))) \Rightarrow (((ap (c_2Ewords_2En2w\ A_{27a}\ A_{27b})\ V0x) = (ap (c_2Ewords_2En2w \\ & A_{27b})\ V1y)) \Leftrightarrow (V0x = (ap (c_2Ewords_2En2w\ A_{27a})\ V1y)))))) \end{aligned}$$