thm_2Ewords_2Ew2w__lt
(TMG6SyLTXqBPSk1z6LfdFJibk8mBprKB4jj)

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Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t)).$

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 6 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 8 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Let $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ebool\_2Eitself\ A0) \tag{2}$$

Let $c\_2Ebool\_2Ethe\_\_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2Ethe\_\_value\ A\_27a \in (\\ ty\_2Ebool\_2Eitself\ A\_27a) \tag{3}$$

Let $c\_2Ewords\_2Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2Edimword\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)} \tag{4}$$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\text{-}2Enum\_2Enum}) \qquad (5)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \qquad (6)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \qquad (7)$$

Definition 9 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num ($

Definition 10 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \land p$
of type $\iota \Rightarrow \iota$.

Definition 11 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Definition 12 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Let $ty\_2Efcp\_2Efinite\_\_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efcp\_2Efinite\_\_image\ A0) \qquad (8)$$

Let $c\_2Efcp\_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Efcp\_2Edimindex\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)})$$
$$(9)$$

Definition 13 *We define* c_2Ebool_2E_3F_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ $c_2Ebool_2E_2F_5C

Definition 14 *We define* c_2Efcp_2Efinite__index *to be* $\lambda A\_27a : \iota.(ap\ ($c_2Emin_2E_40 $(A\_27a^{ty\_2Enum\_2Enu}$

Let $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efcp\_2Ecart$$
$$A0\ A1)$$
$$(10)$$

Let $c\_2Efcp\_2Edest\_\_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efcp\_2Edest\_\_cart$$
$$A\_27a\ A\_27b \in ((A\_27a^{(ty\_2Efcp\_2Efinite\_\_image\ A\_27b)})^{(ty\_2Efcp\_2Ecart\ A\_27a\ A\_27b)})$$
$$(11)$$

Definition 15 *We define* c_2Efcp_2Efcp__index *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in (ty\_2Efcp\_2Ecart\ A\_27$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \qquad (12)$$

Definition 16 *We define* c_2Enum_2E0 *to be* (ap c_2Enum_2EABS__num c_2Enum_2EZERO__REP).

Definition 17 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
(13)

Definition 18 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.$(ap (ap c_2Earithmetic

Definition 19 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
(14)

Definition 20 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 21 *We define* c_2Ebit_2ESBIT *to be* $\lambda V0b \in 2.\lambda V1n \in ty\_2Enum\_2Enum.$(ap (ap (ap (c_2Eboo

Let $c\_2Esum\_num\_2ESUM : \iota$ be given. Assume the following.

$$c\_2Esum\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})^{ty\_2Enum\_2Enum})$$
(15)

Definition 22 *We define* c_2Ewords_2Ew2n *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).$(ap (ap c

Definition 23 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.$(ap (ap c_2Earithmetic

Let $c\_2Earithmetic\_2EDIV : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
(16)

Definition 24 *We define* c_2Ebit_2EDIV__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
(17)

Let $c\_2Earithmetic\_2EMOD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
(18)

Definition 25 *We define* c_2Ebit_2EMOD__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2E$

Definition 26 *We define* c_2Ebit_2EBITS *to be* $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V$

Definition 27 *We define* c_2Ebit_2EBIT *to be* $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$(ap

Definition 28 *We define* c_2Efcp_2EFCP *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(ap$

Definition 29 *We define* c_2Ewords_2En2w *to be* $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.(ap$ (c_2Efcp_2EFC

Definition 30 *We define* c_2Ewords_2Ew2w *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart \; 2 \; A\_27a$

Definition 31 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$\forall V2p \in ty\_2Enum\_2Enum.(((p \; (ap \; (ap \; \text{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0m) \; V1n))\wedge(p \; (ap \; (ap \; \text{c\_2Eprim\_\_rec\_2E\_3C} \; V1n) \; V2p)))\Rightarrow(p \; (ap \; (ap$$
$$\text{c\_2Eprim\_\_rec\_2E\_3C} \; V0m) \; V2p))))))$$

(19)

Assume the following.

$$True \tag{20}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Rightarrow(p \; V0t))\Leftrightarrow(p \; V0t))\wedge((((p \; V0t)\Rightarrow True)\Leftrightarrow$$
$$True)\wedge(((False\Rightarrow(p \; V0t))\Leftrightarrow True)\wedge((((p \; V0t)\Rightarrow(p \; V0t))\Leftrightarrow True)\wedge((\tag{21}$$
$$(p \; V0t)\Rightarrow False)\Leftrightarrow(\neg(p \; V0t)))))))))$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p \; V0t)))\Leftrightarrow(p \; V0t)))\wedge(((\neg True)\Leftrightarrow False)\wedge$$
$$((\neg False)\Leftrightarrow True))) \tag{22}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Leftrightarrow(p \; V0t))\Leftrightarrow(p \; V0t))\wedge((((p \; V0t)\Leftrightarrow True)\Leftrightarrow$$
$$(p \; V0t))\wedge(((False\Leftrightarrow(p \; V0t))\Leftrightarrow(\neg(p \; V0t)))\wedge(((p \; V0t)\Leftrightarrow False)\Leftrightarrow(\neg(\tag{23}$$
$$p \; V0t)))))))$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p \; V0t)))\Leftrightarrow(p \; V0t))) \tag{24}$$

Assume the following.

$$(\forall V0A \in 2.((p \; V0A)\Rightarrow((\neg(p \; V0A))\Rightarrow False))) \tag{25}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \; V0A)\vee(p \; V1B)))\Rightarrow False)\Leftrightarrow$$
$$(((p \; V0A)\Rightarrow False)\Rightarrow((\neg(p \; V1B))\Rightarrow False))))) \tag{26}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p \; V0A))\vee(p \; V1B)))\Rightarrow False)\Leftrightarrow$$
$$((p \; V0A)\Rightarrow((\neg(p \; V1B))\Rightarrow False))))) \tag{27}$$

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Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{28}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow($$
$$(p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg($$
$$p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee$$
$$((\neg(p\ V1q))\vee(\neg(p\ V0p)))))))))))) \tag{29}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow($$
$$(p\ V1q)\wedge(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((\neg(p\ V1q))\vee(\neg(p\ V2r))))\wedge(((p\ V1q)\vee$$
$$(\neg(p\ V0p)))\wedge((p\ V2r)\vee(\neg(p\ V0p))))))))))) \tag{30}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow($$
$$(p\ V1q)\vee(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(\neg(p\ V1q)))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge$$
$$((p\ V1q)\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))))) \tag{31}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow($$
$$(p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((\tag{32}$$
$$\neg(p\ V1q))\vee((p\ V2r)\vee(\neg(p\ V0p))))))))))))$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee$$
$$(p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p)))))))) \tag{33}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(p\ V0p)))) \tag{34}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{35}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0w \in (ty\_2Efcp\_2Ecart$$
$$2\ A\_27a).(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ (\mathsf{c\_2Ewords\_2Ew2n}$$
$$A\_27a)\ V0w))\ (ap\ (c\_2Ewords\_2Edimword\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value$$
$$A\_27a))))) \tag{36}$$

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Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}($$
$$\forall V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$(ap\ (\mathsf{c\_2Ewords\_2Ew2n}\ A\_27b)\ (ap\ (\mathsf{c\_2Ewords\_2Ew2w}\ A\_27a\ A\_27b)$$
$$V0w)))\ (ap\ (\mathsf{c\_2Ewords\_2Ew2n}\ A\_27a)\ V0w))))$$

$$(37)$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}($$
$$\forall V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$(ap\ (\mathsf{c\_2Ewords\_2Ew2n}\ A\_27b)\ (ap\ (\mathsf{c\_2Ewords\_2Ew2w}\ A\_27a\ A\_27b)$$
$$V0w)))\ (ap\ (c\_2Ewords\_2Edimword\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value$$
$$A\_27a)))))$$