

thm_2Ewords_2Eword__L2
(TMXTsE2vMfazES6wUg7bxFGdUvNgJw5PMZw)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \tag{2}$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \tag{4}$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega)^{ty_2Enum_2Enum} \tag{5}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega)^{\omega} \tag{6}$$

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define c_2Ebool_2E21 to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap\ (ap\ (c_2Emin_2E3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V1x \in 2.V1x))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num\ (c_2Enum_2ESUC_REP\ m))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (7)$$

Definition 7 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B) V0n)$.

Definition 8 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (8)$$

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (9)$$

Definition 9 We define $c_2Ebit_2EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.V1n$.

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (10)$$

Definition 10 We define $c_2Ebit_2EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.V1n$.

Definition 11 We define c_2Ebit_2EBITS to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.V1l$.

Let $ty_2EfcP_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2EfcP_2Efinite_image A0) \quad (11)$$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ebool_2Eitself A0) \quad (12)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ebool_2Ethe_value A_27a \in (ty_2Ebool_2Eitself A_27a) \quad (13)$$

Let $c_2EfcP_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2EfcP_2Edimindex A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself A_27a)}) \quad (14)$$

Definition 12 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21) 2) (\lambda V0t \in 2.V0t)$.

Definition 13 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 14 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_Emin_2E_3D_3D_3E V0t) c_Ebool_2E$

Definition 15 We define $c_Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21 2) (\lambda V2t \in$

Definition 16 We define $c_Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge$
of type $\iota \Rightarrow \iota$.

Definition 17 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_Emin_2E_40$

Definition 18 We define $c_Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 19 We define $c_Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap c_Ebool_2E_2F_5C$

Definition 20 We define $c_Efcp_2Efinite_index$ to be $\lambda A_27a : \iota.(ap (c_Emin_2E_40 (A_27a^{ty_2Enum_2Enum}$

Let $ty_2Efcp_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Efcp_2Ecart A0 A1) \quad (15)$$

Let $c_2Efcp_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efcp_2Edest_cart A_27a A_27b \in ((A_27a^{(ty_2Efcp_2Efinite_image A_27b)})^{(ty_2Efcp_2Ecart A_27a A_27b)}) \quad (16)$$

Definition 21 We define $c_Efcp_2Efcp_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efcp_2Ecart A_27a$

Definition 22 We define $c_Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 23 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21 2) (\lambda V2t \in$

Definition 24 We define $c_Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2$

Definition 25 We define $c_2Enumeral_2EiiSUC$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap c_2Enum_2ESUC (ap$

Definition 26 We define $c_Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (17)$$

Let $c_2Enumeral_2Eoncount : \iota$ be given. Assume the following.

$$c_2Enumeral_2Eoncount \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (18)$$

Let $c_2Enumeral_2Eexactlog : \iota$ be given. Assume the following.

$$c_2Enumeral_2Eexactlog \in (ty_2Enum_2Enum^{ty_2Enum_2Enum}) \quad (19)$$

Definition 27 We define c_Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 28 We define $c_Eprim_rec_EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap (ap (ap (c_Ebool_2E$

Definition 29 We define $c_Earithmetic_2EDIV2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_Earithmetic_2E$

Let $c_2Enumeral_2Eexp_help : \iota$ be given. Assume the following.

$$c_2Enumeral_2Eexp_help \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (20)$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \quad (21)$$

Definition 30 We define c_Ebool_2ELET to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in (A_27b^{A_27a}).(\lambda V1x \in A_27$

Definition 31 We define $c_2Enumeral_2EiDUB$ to be $\lambda V0x \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E$

Definition 32 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (22)$$

Definition 33 We define $c_2Enumeral_2Einternal_mult$ to be $c_2Earithmetic_2E_2A$.

Let $c_2Ewords_2Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ewords_2Edimword A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself A_27a)}) \quad (23)$$

Definition 34 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E$

Definition 35 We define c_2Ebit_2EBIT to be $\lambda V0b \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap$

Definition 36 We define c_2Efcf_2EFCF to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap$

Let $c_2Ewords_2EINT_MIN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ewords_2EINT_MIN A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself A_27a)}) \quad (24)$$

Definition 37 We define $c_2Ewords_2En2w$ to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.(ap (c_2Efcf_2EFCF$

Definition 38 We define $c_2Ewords_2Eword_L$ to be $\lambda A_27a : \iota.(ap (c_2Ewords_2En2w A_27a) (ap (c_2E$

Definition 39 We define c_2Ebit_2ESBIT to be $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap (ap (ap (c_2E$

Let $c_2Esum_num_2ESUM : \iota$ be given. Assume the following.

$$c_2Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})^{ty_2Enum_2Enum}) \quad (25)$$

Definition 40 We define c_Ewords_2Ew2n to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a).(ap\ (ap\ c_2E$

Definition 41 We define $c_Ewords_2Eword_mul$ to be $\lambda A_27a : \iota.\lambda V0v \in (ty_2EfcP_2Ecart\ 2\ A_27a).\lambda V$

Definition 42 We define $c_Ewords_2Eword_L2$ to be $\lambda A_27a : \iota.(ap\ (ap\ (c_2Ewords_2Eword_mul\ A_27a)\ ($

Assume the following.

$$\begin{aligned} & ((ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)) = \\ & \quad (ap\ c_2Enum_2ESUC\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1 \\ & \quad \quad c_2Earithmetic_2EZERO)))) \end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m) \\ & \quad c_2Enum_2E0) = V0m)) \end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & \quad ((ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Enum_2E0)\ V0m) = V0m) \wedge (((ap\ (\\ & \quad ap\ c_2Earithmetic_2E_2B\ V0m)\ c_2Enum_2E0) = V0m) \wedge (((ap\ (ap\ c_2Earithmetic_2E_2B \\ & \quad (ap\ c_2Enum_2ESUC\ V0m))\ V1n) = (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\ & \quad \quad V0m)\ V1n))) \wedge ((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ (ap\ c_2Enum_2ESUC \\ & \quad \quad V1n)) = (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n))))))) \end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & \quad (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n) = (ap\ (ap\ c_2Earithmetic_2E_2B \\ & \quad \quad V1n)\ V0m)))) \end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & \quad (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n) = (ap\ (ap\ c_2Earithmetic_2E_2B \\ & \quad \quad V1n)\ V0m)))) \end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & \quad \forall V2p \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m) \\ & \quad (ap\ (ap\ c_2Earithmetic_2E_2B\ V1n)\ V2p)) = (ap\ (ap\ c_2Earithmetic_2E_2B \\ & \quad \quad (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n))\ V2p)))) \end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & \quad (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0m)\ V1n)) \Leftrightarrow (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D \\ & \quad \quad (ap\ c_2Enum_2ESUC\ V0m))\ V1n)))) \end{aligned} \tag{32}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(p (ap (ap c_2Earithmetic_2E_3C_3D c_2Enum_2E0) V0n))) \quad (33)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\neg(p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V0m)))))) \quad (34)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(((ap (ap c_2Earithmetic_2E_2D c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge ((ap (ap c_2Earithmetic_2E_2D V0m) c_2Enum_2E0) = V0m))) \quad (35)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A V0m) V1n)))))))))) \quad (36)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in ty_2Enum_2Enum.(((ap (ap c_2Earithmetic_2E_2A V2p) (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) = (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V2p) V0m)) (ap (ap c_2Earithmetic_2E_2A V2p) V1n)))))) \quad (37)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in ty_2Enum_2Enum.(((ap (ap c_2Earithmetic_2E_2A V0m) (ap (ap c_2Earithmetic_2E_2A V1n) V2p)) = (ap (ap c_2Earithmetic_2E_2A (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) V2p)))))) \quad (38)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad \forall V2p \in ty_2Enum_2Enum. (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))) \Rightarrow (p (\\
& \quad \quad ap (ap c_2Earithmetic_2E_3C_3D V0m) V2p))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad (p (ap (ap c_2Eprim_rec_2E_3C V1n) V0m)) \Rightarrow (\exists V2p \in ty_2Enum_2Enum. \\
& \quad (V0m = (ap (ap c_2Earithmetic_2E_2B V1n) (ap (ap c_2Earithmetic_2E_2B \\
& \quad \quad V2p) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad \quad \quad c_2Earithmetic_2EZERO))))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in ty_2Enum_2Enum. (\forall V1q \in ty_2Enum_2Enum. (\\
& \quad \forall V2n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2EEXP V2n) \\
& \quad (ap (ap c_2Earithmetic_2E_2B V0p) V1q)) = (ap (ap c_2Earithmetic_2E_2A \\
& \quad (ap (ap c_2Earithmetic_2EEXP V2n) V0p)) (ap (ap c_2Earithmetic_2EEXP \\
& \quad \quad V2n) V1q))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) \\
& \quad V0n) = (ap (ap c_2Earithmetic_2E_2B V0n) V0n)))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0c \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2D V0c) \\
& \quad V0c) = c_2Enum_2E0))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad ((ap (ap c_2Earithmetic_2E_2A V0m) V1n) = c_2Enum_2E0) \Leftrightarrow ((V0m = \\
& \quad \quad c_2Enum_2E0) \vee (V1n = c_2Enum_2E0))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad (V0m = V1n) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \wedge (p (\\
& \quad \quad ap (ap c_2Earithmetic_2E_3C_3D V1n) V0m))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) (ap (ap c_2Earithmetic_2E_2B \\
& V0m) V2p))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (\neg (p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap c_2Enum_2ESUC V1n)) V0m))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (\neg (V0m = V1n)) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC \\
& V0m)) V1n)) \vee (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC \\
& V1n)) V0m))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC V0n) = (ap (ap \\
& c_2Earithmetic_2E_2B (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO))) V0n)))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0m) V1n)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D (ap (ap c_2Earithmetic_2E_2A \\
& (ap c_2Enum_2ESUC V2p)) V0m)) (ap (ap c_2Earithmetic_2E_2A (ap \\
& c_2Enum_2ESUC V2p)) V1n))))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0k \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2EMOD V0k) \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = \\
& c_2Enum_2E0))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1k \in ty_2Enum_2Enum. (\\
& (p (ap (ap c_2Eprim_rec_2E_3C V1k) V0n)) \Rightarrow ((ap (ap c_2Earithmetic_2EMOD \\
& V1k) V0n) = V1k)))
\end{aligned} \tag{52}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V0n)) \Rightarrow (\forall V1k \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EMOD (ap (ap c_2Earithmetic_2E_2A V1k) V0n)) V0n) = c_2Enum_2E0)))))) \quad (53)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V0n)) \Rightarrow ((ap (ap c_2Earithmetic_2EMOD c_2Enum_2E0) V0n) = c_2Enum_2E0))) \quad (54)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\forall V1p \in ty_2Enum_2Enum.(\forall V2q \in ty_2Enum_2Enum.(((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V0n)) \wedge (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V2q))) \Rightarrow ((ap (ap c_2Earithmetic_2E_2A V0n) (ap (ap c_2Earithmetic_2EMOD V1p) V2q))) = (ap (ap c_2Earithmetic_2EMOD (ap (ap c_2Earithmetic_2E_2A V0n) V1p)) (ap (ap c_2Earithmetic_2E_2A V0n) V2q))))))) \quad (55)$$

Assume the following.

$$(\forall V0P \in (2^{ty_2Enum_2Enum}).(\forall V1a \in ty_2Enum_2Enum.(\forall V2b \in ty_2Enum_2Enum.((p (ap V0P (ap (ap c_2Earithmetic_2E_2D V1a) V2b))) \Leftrightarrow (\forall V3d \in ty_2Enum_2Enum.(((V2b = (ap (ap c_2Earithmetic_2E_2B V1a) V3d)) \Rightarrow (p (ap V0P c_2Enum_2E0)) \wedge ((V1a = (ap (ap c_2Earithmetic_2E_2B V2b) V3d)) \Rightarrow (p (ap V0P V3d)))))))))) \quad (56)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\forall V1m \in ty_2Enum_2Enum.(((ap (ap c_2Earithmetic_2EEXP V0n) V1m) = c_2Enum_2E0) \Leftrightarrow ((V0n = c_2Enum_2E0) \wedge (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V1m)))))) \quad (57)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0n) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) \wedge ((ap (ap c_2Earithmetic_2EEXP V0n) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0n))) \quad (58)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) V0n)))) \quad (59)$$

Assume the following.

$$\begin{aligned}
& (\forall V0h \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (ap (ap (ap c_2Ebit_2EBITS V0h) c_2Enum_2E0) V1n) = (ap (ap c_2Earithmetic_2EMOD \\
& V1n) (ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) (ap c_2Enum_2ESUC \\
& V0h))))))
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& (\forall V0h \in ty_2Enum_2Enum. (\forall V1l \in ty_2Enum_2Enum. (\\
& \forall V2a \in ty_2Enum_2Enum. (\forall V3b \in ty_2Enum_2Enum. (\\
& \forall V4x \in ty_2Enum_2Enum. (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V1l) V4x)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V4x) V0h))) \Rightarrow ((p \\
& (ap (ap c_2Ebit_2EBIT V4x) V2a)) \Leftrightarrow (p (ap (ap c_2Ebit_2EBIT V4x) V3b)))))) \Leftrightarrow \\
& ((ap (ap (ap c_2Ebit_2EBITS V0h) V1l) V2a) = (ap (ap (ap c_2Ebit_2EBITS \\
& V0h) V1l) V3b))))))
\end{aligned} \tag{61}$$

Assume the following.

$$True \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
& V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))))
\end{aligned} \tag{63}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \tag{64}$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee \neg(p V0t))) \tag{65}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in \\
A_27a. (p V0t)) \Leftrightarrow (p V0t))) \tag{66}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \wedge \\
& ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3))))))
\end{aligned} \tag{67}$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow \neg(p V0t))) \tag{68}$$

Assume the following.

$$(\forall V0t \in 2. (\neg(p V0t) \Rightarrow ((p V0t) \Rightarrow False))) \tag{69}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (70)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\
& (p V0t)) \Leftrightarrow (p V0t)))))) \quad (71)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\
& (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (72)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True))) \quad (73)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow \\
& True)) \quad (74)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in \\
& A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (75)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& p V0t)))))) \quad (76)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0t1 \in A.27a.(\forall V1t2 \in \\
& A.27a.(((ap (ap (ap (c.2Ebool.2ECOND A.27a) c.2Ebool.2ET) V0t1) \\
& V1t2) = V0t1) \wedge ((ap (ap (ap (c.2Ebool.2ECOND A.27a) c.2Ebool.2EF \\
& V0t1) V1t2) = V1t2)))))) \quad (77)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\forall V1x \in \\
& A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p (ap V0P V2x)))))) \quad (78)
\end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ & 2.(((\forall V2x \in A.27a.(p \ (ap \ V0P \ V2x))) \wedge (p \ V1Q))) \Leftrightarrow (\forall V3x \in \\ & A.27a.((p \ (ap \ V0P \ V3x)) \wedge (p \ V1Q)))))) \end{aligned} \quad (79)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ & 2.(((\exists V2x \in A.27a.(p \ (ap \ V0P \ V2x))) \vee (p \ V1Q))) \Leftrightarrow (\exists V3x \in \\ & A.27a.((p \ (ap \ V0P \ V3x)) \vee (p \ V1Q)))))) \end{aligned} \quad (80)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (\\ & 2^{A.27a}).((\exists V2x \in A.27a.((p \ V0P) \wedge (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow ((p \\ & V0P) \wedge (\exists V3x \in A.27a.(p \ (ap \ V1Q \ V3x)))))) \end{aligned} \quad (81)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (\\ & 2^{A.27a}).((\forall V2x \in A.27a.((p \ (ap \ V1P \ V2x)) \vee (p \ V0Q))) \Leftrightarrow ((\forall V3x \in \\ & A.27a.(p \ (ap \ V1P \ V3x)) \vee (p \ V0Q)))))) \end{aligned} \quad (82)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V0A) \vee (\\ & (p \ V1B) \vee (p \ V2C))) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \vee (p \ V2C)))))) \end{aligned} \quad (83)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((p \ V0A) \vee (p \ V1B)) \Leftrightarrow ((p \ V1B) \vee \\ & (p \ V0A)))))) \end{aligned} \quad (84)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \wedge (p \ V1B))) \Leftrightarrow ((\neg(\\ & p \ V0A) \vee (\neg(p \ V1B)))))) \wedge (((\neg((p \ V0A) \vee (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A) \wedge (\neg(p \ V1B))))))))) \end{aligned} \quad (85)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((p \ V0A) \Rightarrow (p \ V1B)) \Leftrightarrow ((\neg(p \ V0A) \vee \\ & (p \ V1B)))))) \end{aligned} \quad (86)$$

Assume the following.

$$(\forall V0t \in 2.(((p \ V0t) \Rightarrow False) \Leftrightarrow ((p \ V0t) \Leftrightarrow False))) \quad (87)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow \\ & ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \end{aligned} \quad (88)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \vee ((\neg(p V0t1)) \wedge (\neg(p V1t2)))))))) \quad (89)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (90)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\ & (\forall V2x \in A_{.27a}.(\forall V3x_{.27} \in A_{.27a}.(\forall V4y \in A_{.27a}. \\ & (\forall V5y_{.27} \in A_{.27a}.(((p V0P) \Leftrightarrow (p V1Q)) \wedge ((p V1Q) \Rightarrow (V2x = V3x_{.27})) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y_{.27})))) \Rightarrow ((ap (ap (ap (c_{.2Ebool_2ECOND} A_{.27a}) \\ & V0P) V2x) V4y) = (ap (ap (ap (c_{.2Ebool_2ECOND} A_{.27a}) V1Q) V3x_{.27}) \\ & V5y_{.27})))))))))) \quad (91) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\\ & \forall V0P \in ((2^{A_{.27b}})^{A_{.27a}}).((\forall V1x \in A_{.27a}.(\exists V2y \in \\ & A_{.27b}.(p (ap (ap V0P V1x) V2y)))) \Leftrightarrow (\exists V3f \in (A_{.27b}^{A_{.27a}}). \\ & \forall V4x \in A_{.27a}.(p (ap (ap V0P V4x) (ap V3f V4x)))))) \quad (92) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\\ & \forall V0x \in (ty_{.2EfcP_2Ecart} A_{.27a} A_{.27b}).(\forall V1y \in (ty_{.2EfcP_2Ecart} \\ & A_{.27a} A_{.27b}).((V0x = V1y) \Leftrightarrow (\forall V2i \in ty_{.2Enum_2Enum}.((p (ap \\ & (ap c_{.2Eprim_rec_2E_3C} V2i) (ap (c_{.2EfcP_2Edimindex} A_{.27b}) (\\ & c_{.2Ebool_2Ethe_value} A_{.27b})))) \Rightarrow ((ap (ap (c_{.2EfcP_2EfcP_index} \\ & A_{.27a} A_{.27b}) V0x) V2i) = (ap (ap (c_{.2EfcP_2EfcP_index} A_{.27a} A_{.27b}) \\ & V1y) V2i)))))) \quad (93) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\\ & \forall V0g \in (A_{.27a}^{ty_{.2Enum_2Enum}}).(\forall V1i \in ty_{.2Enum_2Enum}. \\ & ((p (ap (ap (ap c_{.2Eprim_rec_2E_3C} V1i) (ap (c_{.2EfcP_2Edimindex} A_{.27b}) \\ & (c_{.2Ebool_2Ethe_value} A_{.27b})))) \Rightarrow ((ap (ap (c_{.2EfcP_2EfcP_index} \\ & A_{.27a} A_{.27b}) (ap (c_{.2EfcP_2EFCP} A_{.27a} A_{.27b}) V0g)) V1i) = (ap V0g \\ & V1i)))))) \quad (94) \end{aligned}$$

Assume the following.

$$\begin{aligned}
 &(((ap\ c_2Enum_2ESUC\ c_2Earithmetic_2EZERO) = (ap\ c_2Earithmetic_2EBIT1 \\
 &\quad c_2Earithmetic_2EZERO)) \wedge (\forall V0n \in ty_2Enum_2Enum. ((ap \\
 &\quad c_2Enum_2ESUC\ (ap\ c_2Earithmetic_2EBIT1\ V0n)) = (ap\ c_2Earithmetic_2EBIT2 \\
 &\quad V0n))) \wedge (\forall V1n \in ty_2Enum_2Enum. ((ap\ c_2Enum_2ESUC\ (ap\ c_2Earithmetic_2EBIT2 \\
 &\quad V1n)) = (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enum_2ESUC\ V1n))))))
 \end{aligned}
 \tag{95}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum. (\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (\\
& ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (\\
& ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (\\
& ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap \\
& c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC \\
& (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ c_2Earithmetic_2EZERO)) = (\\
& ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (\\
& ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT1 \\
& V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m))))))))))))))))))))))))))))))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((c_2Earithmic_2EZERO = (ap\ c_2Earithmic_2EBIT1\ V0n)) \Leftrightarrow False) \wedge \\
& (((ap\ c_2Earithmic_2EBIT1\ V0n) = c_2Earithmic_2EZERO) \Leftrightarrow \\
& False) \wedge (((c_2Earithmic_2EZERO = (ap\ c_2Earithmic_2EBIT2 \\
& V0n)) \Leftrightarrow False) \wedge (((ap\ c_2Earithmic_2EBIT2\ V0n) = c_2Earithmic_2EZERO) \Leftrightarrow \\
& False) \wedge (((ap\ c_2Earithmic_2EBIT1\ V0n) = (ap\ c_2Earithmic_2EBIT2 \\
& V1m)) \Leftrightarrow False) \wedge (((ap\ c_2Earithmic_2EBIT2\ V0n) = (ap\ c_2Earithmic_2EBIT1 \\
& V1m)) \Leftrightarrow False) \wedge (((ap\ c_2Earithmic_2EBIT1\ V0n) = (ap\ c_2Earithmic_2EBIT1 \\
& V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap\ c_2Earithmic_2EBIT2\ V0n) = (ap\ c_2Earithmic_2EBIT2 \\
& V1m)) \Leftrightarrow (V0n = V1m))))))))) \\
\end{aligned} \tag{98}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ c_2Earithmic_2EZERO)\ (ap\ c_2Earithmic_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ c_2Earithmic_2EZERO) \\
& (ap\ c_2Earithmic_2EBIT2\ V0n))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& V0n)\ c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& (ap\ c_2Earithmic_2EBIT1\ V0n))\ (ap\ c_2Earithmic_2EBIT1\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& (ap\ c_2Earithmic_2EBIT2\ V0n))\ (ap\ c_2Earithmic_2EBIT2\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& (ap\ c_2Earithmic_2EBIT1\ V0n))\ (ap\ c_2Earithmic_2EBIT2\ V1m))) \Leftrightarrow \\
& (\neg(p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V1m)\ V0n)))) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& (ap\ c_2Earithmic_2EBIT2\ V0n))\ (ap\ c_2Earithmic_2EBIT1\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ V1m))))))))) \\
\end{aligned} \tag{99}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ c_2Earithmic_2EZERO)\ V0n)) \Leftrightarrow \\
& True) \wedge (((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ (ap\ c_2Earithmic_2EBIT1 \\
& V0n))\ c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D \\
& (ap\ c_2Earithmic_2EBIT2\ V0n))\ c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ (ap\ c_2Earithmic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmic_2EBIT1\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D \\
& V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ (ap\ c_2Earithmic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmic_2EBIT2\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D \\
& V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ (ap\ c_2Earithmic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmic_2EBIT1\ V1m))) \Leftrightarrow (\neg(p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D \\
& V1m)\ V0n)))) \wedge (((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ (ap\ c_2Earithmic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmic_2EBIT2\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D \\
& V0n)\ V1m))))))))) \\
\end{aligned} \tag{100}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (((ap\ c_2Enumeral_2EiDUB\ (ap\ c_2Earithmetic_2EBIT1\ V0n)) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enumeral_2EiDUB\ V0n))) \wedge \\
& \quad (((ap\ c_2Enumeral_2EiDUB\ (ap\ c_2Earithmetic_2EBIT2\ V0n)) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Earithmetic_2EBIT1\ V0n))) \wedge ((ap\ c_2Enumeral_2EiDUB\ c_2Earithmetic_2EZERO) = c_2Earithmetic_2EZERO)))) \\
& \hspace{15em} (101)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((ap\ (ap\ c_2Earithmetic_2E_2A\ c_2Earithmetic_2EZERO)\ V0n) = c_2Earithmetic_2EZERO) \wedge \\
& \quad (((ap\ (ap\ c_2Earithmetic_2E_2A\ V0n)\ c_2Earithmetic_2EZERO) = \\
& \quad c_2Earithmetic_2EZERO) \wedge (((ap\ (ap\ c_2Earithmetic_2E_2A\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ V1m) = (ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Enumeral_2EiDUB\ (ap\ (ap\ c_2Earithmetic_2E_2A\ V0n)\ V1m)))\ V1m))) \wedge ((ap\ (ap\ c_2Earithmetic_2E_2A\ (ap\ c_2Earithmetic_2EBIT2\ V0n))\ V1m) = (ap\ c_2Enumeral_2EiDUB\ (ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ (ap\ c_2Earithmetic_2E_2A\ V0n)\ V1m))\ V1m)))))))))) \\
& \hspace{15em} (102)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((p\ (ap\ c_2Earithmetic_2EEVEN\ c_2Earithmetic_2EZERO)) \wedge \\
& \quad ((p\ (ap\ c_2Earithmetic_2EEVEN\ (ap\ c_2Earithmetic_2EBIT2\ V0n))) \wedge \\
& \quad ((\neg(p\ (ap\ c_2Earithmetic_2EEVEN\ (ap\ c_2Earithmetic_2EBIT1\ V0n)))) \wedge \\
& \quad \quad ((\neg(p\ (ap\ c_2Earithmetic_2EODD\ c_2Earithmetic_2EZERO))) \wedge ((\neg(p\ (ap\ c_2Earithmetic_2EODD\ (ap\ c_2Earithmetic_2EBIT2\ V0n)))) \wedge \\
& \quad \quad (p\ (ap\ c_2Earithmetic_2EODD\ (ap\ c_2Earithmetic_2EBIT1\ V0n)))))))))) \\
& \hspace{15em} (103)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (((ap\ (ap\ c_2Earithmetic_2EEXP\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)))\ c_2Enum_2E0) = (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))) \wedge (((ap\ (ap\ c_2Earithmetic_2EEXP\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)))\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ V0n))) = (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ (ap\ c_2Enumeral_2Eexp_help\ (ap\ c_2Eprim_rec_2EPRE\ (ap\ c_2Earithmetic_2EBIT1\ V0n)))\ c_2Earithmetic_2EZERO)))) \wedge \\
& \quad ((ap\ (ap\ c_2Earithmetic_2EEXP\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)))\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ V0n))) = (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ (ap\ c_2Enumeral_2Eexp_help\ (ap\ c_2Earithmetic_2EBIT1\ V0n)))\ c_2Earithmetic_2EZERO)))))) \\
& \hspace{15em} (104)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0x \in ty_2Enum_2Enum.((ap (ap c_2Enumeral_2Eonecount \\
& c_2Earithmetic_2EZERO) V0x) = V0x)) \wedge ((\forall V1n \in ty_2Enum_2Enum. \\
& (\forall V2x \in ty_2Enum_2Enum.((ap (ap c_2Enumeral_2Eonecount \\
& (ap c_2Earithmetic_2EBIT1 V1n)) V2x) = (ap (ap c_2Enumeral_2Eonecount \\
& V1n) (ap c_2Enum_2ESUC V2x)))))) \wedge ((\forall V3n \in ty_2Enum_2Enum. \\
& (\forall V4x \in ty_2Enum_2Enum.((ap (ap c_2Enumeral_2Eonecount \\
& (ap c_2Earithmetic_2EBIT2 V3n)) V4x) = c_2Earithmetic_2EZERO))))))
\end{aligned} \tag{105}$$

Assume the following.

$$\begin{aligned}
& (((ap c_2Enumeral_2Eexactlog c_2Earithmetic_2EZERO) = c_2Earithmetic_2EZERO) \wedge \\
& ((\forall V0n \in ty_2Enum_2Enum.((ap c_2Enumeral_2Eexactlog (\\
& ap c_2Earithmetic_2EBIT1 V0n)) = c_2Earithmetic_2EZERO)) \wedge ((\forall V1n \in \\
& ty_2Enum_2Enum.((ap c_2Enumeral_2Eexactlog (ap c_2Earithmetic_2EBIT2 \\
& V1n)) = (ap (ap (c_2Ebool_2ELET ty_2Enum_2Enum ty_2Enum_2Enum) \\
& (\lambda V2x \in ty_2Enum_2Enum.(ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) \\
& (ap (ap (c_2Emin_2E_3D ty_2Enum_2Enum) V2x) c_2Earithmetic_2EZERO)) \\
& c_2Earithmetic_2EZERO) (ap c_2Earithmetic_2EBIT1 V2x)))) (ap \\
& (ap c_2Enumeral_2Eonecount V1n) c_2Earithmetic_2EZERO))))))
\end{aligned} \tag{106}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1x \in ty_2Enum_2Enum. (\\
& \forall V2y \in ty_2Enum_2Enum. (((ap (ap c_2Earithmic_2E_2A c_2Earithmic_2EZERO) \\
& V0n) = c_2Earithmic_2EZERO) \wedge (((ap (ap c_2Earithmic_2E_2A \\
& V0n) c_2Earithmic_2EZERO) = c_2Earithmic_2EZERO) \wedge ((ap \\
& (ap c_2Earithmic_2E_2A (ap c_2Earithmic_2EBIT1 V1x)) (ap \\
& c_2Earithmic_2EBIT1 V2y)) = (ap (ap c_2Enumeral_2Einternal_mult \\
& (ap c_2Earithmic_2EBIT1 V1x)) (ap c_2Earithmic_2EBIT1 V2y)))) \wedge \\
& (((ap (ap c_2Earithmic_2E_2A (ap c_2Earithmic_2EBIT1 V1x)) \\
& (ap c_2Earithmic_2EBIT2 V2y)) = (ap (ap (c_2Ebool_2ELET ty_2Enum_2Enum \\
& ty_2Enum_2Enum) (\lambda V3n \in ty_2Enum_2Enum. (ap (ap (ap (c_2Ebool_2ECOND \\
& ty_2Enum_2Enum) (ap c_2Earithmic_2EODD V3n)) (ap (ap c_2Enumeral_2Eexp_help \\
& (ap c_2Earithmic_2EDIV2 V3n)) (ap c_2Eprim_rec_2EPRE (ap c_2Earithmic_2EBIT1 \\
& V1x)))) (ap (ap c_2Enumeral_2Einternal_mult (ap c_2Earithmic_2EBIT1 \\
& V1x)) (ap c_2Earithmic_2EBIT2 V2y)))))) (ap c_2Enumeral_2Eexactlog \\
& (ap c_2Earithmic_2EBIT2 V2y)))) \wedge (((ap (ap c_2Earithmic_2E_2A \\
& (ap c_2Earithmic_2EBIT2 V1x)) (ap c_2Earithmic_2EBIT1 V2y)) = \\
& (ap (ap (c_2Ebool_2ELET ty_2Enum_2Enum ty_2Enum_2Enum) (\lambda V4m \in \\
& ty_2Enum_2Enum. (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) \\
& (ap c_2Earithmic_2EODD V4m)) (ap (ap c_2Enumeral_2Eexp_help \\
& (ap c_2Earithmic_2EDIV2 V4m)) (ap c_2Eprim_rec_2EPRE (ap c_2Earithmic_2EBIT1 \\
& V2y)))) (ap (ap c_2Enumeral_2Einternal_mult (ap c_2Earithmic_2EBIT2 \\
& V1x)) (ap c_2Earithmic_2EBIT1 V2y)))))) (ap c_2Enumeral_2Eexactlog \\
& (ap c_2Earithmic_2EBIT2 V1x)))) \wedge ((ap (ap c_2Earithmic_2E_2A \\
& (ap c_2Earithmic_2EBIT2 V1x)) (ap c_2Earithmic_2EBIT2 V2y)) = \\
& (ap (ap (c_2Ebool_2ELET ty_2Enum_2Enum ty_2Enum_2Enum) (\lambda V5m \in \\
& ty_2Enum_2Enum. (ap (ap (c_2Ebool_2ELET ty_2Enum_2Enum ty_2Enum_2Enum) \\
& (\lambda V6n \in ty_2Enum_2Enum. (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) \\
& (ap c_2Earithmic_2EODD V5m)) (ap (ap c_2Enumeral_2Eexp_help \\
& (ap c_2Earithmic_2EDIV2 V5m)) (ap c_2Eprim_rec_2EPRE (ap c_2Earithmic_2EBIT2 \\
& V2y)))) (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) (ap c_2Earithmic_2EODD \\
& V6n)) (ap (ap c_2Enumeral_2Eexp_help (ap c_2Earithmic_2EDIV2 \\
& V6n)) (ap c_2Eprim_rec_2EPRE (ap c_2Earithmic_2EBIT2 V1x)))) \\
& (ap (ap c_2Enumeral_2Einternal_mult (ap c_2Earithmic_2EBIT2 \\
& V1x)) (ap c_2Earithmic_2EBIT2 V2y)))))) (ap c_2Enumeral_2Eexactlog \\
& (ap c_2Earithmic_2EBIT2 V2y)))))) (ap c_2Enumeral_2Eexactlog \\
& (ap c_2Earithmic_2EBIT2 V1x))))))))) (107)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((ap (ap c_2Enumeral_2Einternal_mult c_2Earithmetic_2EZERO) \\
V0n) = c_2Earithmetic_2EZERO) \wedge ((ap (ap c_2Enumeral_2Einternal_mult \\
& V0n) c_2Earithmetic_2EZERO) = c_2Earithmetic_2EZERO) \wedge ((ap \\
& (ap c_2Enumeral_2Einternal_mult (ap c_2Earithmetic_2EBIT1 \\
V0n)) V1m) = (ap c_2Enumeral_2EiZ (ap (ap c_2Earithmetic_2E_2B \\
& (ap c_2Enumeral_2EiDUB (ap (ap c_2Enumeral_2Einternal_mult \\
& V0n) V1m))) V1m))) \wedge ((ap (ap c_2Enumeral_2Einternal_mult (ap \\
& c_2Earithmetic_2EBIT2 V0n)) V1m) = (ap c_2Enumeral_2EiDUB (ap \\
& c_2Enumeral_2EiZ (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Enumeral_2Einternal_mult \\
& V0n) V1m))) V1m)))))))))
\end{aligned} \tag{108}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{109}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{110}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{111}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{112}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \tag{113}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{114}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{115}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee \neg(p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee \neg(p V0p)))))))))) \\
& \tag{116}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge ((\\
& \neg(p V1q) \vee ((p V2r) \vee \neg(p V0p)))))))))) \\
& \tag{117}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow \neg(p V1q)) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge (\neg(p V1q) \vee \neg(p V0p)))))) \\
& \tag{118}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow ((ap (c_2Ewords_2Edimword A_27a) \\
& (c_2Ebool_2Ethe_value A_27a)) = (ap (ap c_2Earithmetic_2EEXP \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) \\
& (ap (c_2Efc_2Edimindex A_27a) (c_2Ebool_2Ethe_value A_27a)))) \\
& \tag{119}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow ((ap (c_2Ewords_2EINT_MIN A_27a) \\
& (c_2Ebool_2Ethe_value A_27a)) = (ap (ap c_2Earithmetic_2EEXP \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) \\
& (ap (ap c_2Earithmetic_2E_2D (ap (c_2Efc_2Edimindex A_27a) (\\
& c_2Ebool_2Ethe_value A_27a))) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \\
& \tag{120}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\exists V0m \in ty_2Enum_2Enum. (\\
& (ap (c_2Efc_2Edimindex A_27a) (c_2Ebool_2Ethe_value A_27a)) = \\
& (ap c_2Enum_2ESUC V0m))) \\
& \tag{121}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0m \in ty_2Enum_2Enum. (\\
& \forall V1n \in ty_2Enum_2Enum. ((ap (ap (c_2Ewords_2Eword_mul \\
& A_27a) (ap (c_2Ewords_2En2w A_27a) V0m)) (ap (c_2Ewords_2En2w \\
& A_27a) V1n)) = (ap (c_2Ewords_2En2w A_27a) (ap (ap c_2Earithmetic_2E_2A \\
& V0m) V1n)))))) \\
& \tag{122}
\end{aligned}$$

Theorem 1

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow ((c_Ewords_Eword_L2\ A_{27a}) =$$
$$(ap\ (ap\ (ap\ (c_Ebool_ECOND\ (ty_Efc_Ecart\ 2\ A_{27a}))\ (ap\ (ap$$
$$c_Eprim_rec_E_3C\ (ap\ c_Earithmetic_ENUMERAL\ (ap\ c_Earithmetic_EBIT1$$
$$c_Earithmetic_EZERO)))\ (ap\ (c_Efc_Edimindex\ A_{27a})\ (c_Ebool_Ethe_value$$
$$A_{27a}))))\ (ap\ (c_Ewords_En2w\ A_{27a})\ c_Eenum_E0))\ (c_Ewords_Eword_L$$
$$A_{27a}))$$