

thm_2Ewords_2Eword__L2__MULT
(TMaL9zZfDZxoHw4EGfFqUqz6tFVtUmsNw8p)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_5C_2E_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 6 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 7 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ebool_2Eitself A0) \quad (1)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ebool_2Ethe_value A_27a \in (ty_2Ebool_2Eitself A_27a) \quad (2)$$

Let $ty_2Eenum_2Eenum : \iota$ be given. Assume the following.

$$nonempty ty_2Eenum_2Eenum \quad (3)$$

Let $c_2Ewords_2EINT_MIN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ewords_2EINT_MIN A_27a \in (ty_2Eenum_2Eenum^{(ty_2Ebool_2Eitself A_27a)}) \quad (4)$$

Let $c_2Eenum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Eenum_2EZERO_REP \in \omega \quad (5)$$

Let $c_2Eenum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Eenum_2EABS_num \in (ty_2Eenum_2Eenum^{\omega}) \quad (6)$$

Definition 8 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 9 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (7)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (8)$$

Definition 10 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (9)$$

Definition 11 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 12 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 13 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (10)$$

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (11)$$

Definition 14 We define $c_2Ebit_2EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (12)$$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (13)$$

Definition 15 We define $c_2Ebit_2EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 16 We define c_2Ebit_2EBITS to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.\lambda V$

Definition 17 We define c_2Ebit_2EBIT to be $\lambda V0b \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap$

Let $ty_2Efc_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Efc_2Efinite_image\ A0) \quad (14)$$

Let $c_2Efc_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Efc_2Edimindex\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (15)$$

Definition 18 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 19 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A \wedge$
of type $\iota \Rightarrow \iota$.

Definition 20 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 21 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 22 We define $c_2Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ c_2Ebool_2E_2F_5C$

Definition 23 We define $c_2Efc_2Efinite_index$ to be $\lambda A_27a : \iota.(ap\ (c_2Emin_2E_40\ (A_27a^{ty_2Enum_2Enum}$

Let $ty_2Efc_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Efc_2Ecart\ A0\ A1) \quad (16)$$

Let $c_2Efc_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efc_2Edest_cart\ A_27a\ A_27b \in ((A_27a^{(ty_2Efc_2Efinite_image\ A_27b)})^{(ty_2Efc_2Ecart\ A_27a\ A_27b)}) \quad (17)$$

Definition 24 We define $c_2Efc_2Efc_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efc_2Ecart\ A_27a\ A_27b)$

Definition 25 We define c_2Efc_2EFCP to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap\ ($

Definition 26 We define $c_2Ewords_2Een2w$ to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.(ap\ (c_2Efc_2EFCP$

Definition 27 We define $c_2Ewords_2Eword_L$ to be $\lambda A_27a : \iota.(ap\ (c_2Ewords_2Een2w\ A_27a)\ (ap\ (c_2Eword$

Definition 28 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 29 We define c_2Ebit_2ESBIT to be $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ (c_2Ebool$

Let $c_2Esum_num_2ESUM : \iota$ be given. Assume the following.

$$c_2Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})^{ty_2Enum_2Enum}) \quad (18)$$

Definition 30 We define $c_2Ewords_2Ew2n$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efc_2Ecart\ 2\ A_27a).(ap\ (ap\ c$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (19)$$

Definition 31 We define $c_2Ewords_2Eword_mul$ to be $\lambda A_27a : \iota. \lambda V0v \in (ty_2Efc_2Ecart_2_A_27a). \lambda V$

Definition 32 We define $c_2Ewords_2Eword_l2$ to be $\lambda A_27a : \iota. (ap (ap (c_2Ewords_2Eword_mul A_27a) ($

Definition 33 We define $c_2Ewords_2Eword_add$ to be $\lambda A_27a : \iota. \lambda V0v \in (ty_2Efc_2Ecart_2_A_27a). \lambda V$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (20)$$

Let $c_2Ewords_2Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Ewords_2Edimword A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself A_27a)}) \quad (21)$$

Definition 34 We define $c_2Ewords_2Eword_2comp$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Efc_2Ecart_2_A_27a).$

Assume the following.

$$True \quad (22)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee \neg(p V0t))) \quad (25)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (26)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (28)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in \\ & A_27a. (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1) \\ & V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF) \\ & V0t1)\ V1t2) = V1t2)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\ & (\forall V5y_27 \in A_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge ((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ & ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a) \\ & V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27) \\ & V5y_27))))))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0t1 \in A_27a. (\forall V1t2 \in \\ & A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1) \\ & V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a. (\forall V3t2 \in A_27a. ((ap \\ & (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V2t1)\ V3t2) = V3t2)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0v \in (ty.2EfcP.2Ecart \\
& \quad 2\ A.27a).(\forall V1w \in (ty.2EfcP.2Ecart\ 2\ A.27a).(((ap\ (ap\ (\\
& \quad c.2Ewords.2Eword_mul\ A.27a)\ (ap\ (c.2Ewords.2En2w\ A.27a)\ c.2Enum.2E0)) \\
V0v) = (ap\ (c.2Ewords.2En2w\ A.27a)\ c.2Enum.2E0)) \wedge (((ap\ (ap\ (c.2Ewords.2Eword_mul \\
& \quad A.27a)\ V0v)\ (ap\ (c.2Ewords.2En2w\ A.27a)\ c.2Enum.2E0)) = (ap\ (c.2Ewords.2En2w \\
& \quad A.27a)\ c.2Enum.2E0)) \wedge (((ap\ (ap\ (c.2Ewords.2Eword_mul\ A.27a) \\
& \quad (ap\ (c.2Ewords.2En2w\ A.27a)\ (ap\ c.2Earithmetic.2ENUMERAL\ (ap \\
& \quad c.2Earithmetic.2EBIT1\ c.2Earithmetic.2EZERO))))\ V0v) = V0v) \wedge \\
& \quad (((ap\ (ap\ (c.2Ewords.2Eword_mul\ A.27a)\ V0v)\ (ap\ (c.2Ewords.2En2w \\
& \quad A.27a)\ (ap\ c.2Earithmetic.2ENUMERAL\ (ap\ c.2Earithmetic.2EBIT1 \\
& \quad c.2Earithmetic.2EZERO)))) = V0v) \wedge (((ap\ (ap\ (c.2Ewords.2Eword_mul \\
& \quad A.27a)\ (ap\ (ap\ (c.2Ewords.2Eword_add\ A.27a)\ V0v)\ (ap\ (c.2Ewords.2En2w \\
& \quad A.27a)\ (ap\ c.2Earithmetic.2ENUMERAL\ (ap\ c.2Earithmetic.2EBIT1 \\
& \quad c.2Earithmetic.2EZERO))))\ V1w) = (ap\ (ap\ (c.2Ewords.2Eword_add \\
& \quad A.27a)\ (ap\ (ap\ (c.2Ewords.2Eword_mul\ A.27a)\ V0v)\ V1w))\ V1w)) \wedge \\
& \quad (((ap\ (ap\ (c.2Ewords.2Eword_mul\ A.27a)\ V0v)\ (ap\ (ap\ (c.2Ewords.2Eword_add \\
& \quad A.27a)\ V1w)\ (ap\ (c.2Ewords.2En2w\ A.27a)\ (ap\ c.2Earithmetic.2ENUMERAL \\
& \quad (ap\ c.2Earithmetic.2EBIT1\ c.2Earithmetic.2EZERO)))) = (ap\ (\\
& \quad ap\ (c.2Ewords.2Eword_add\ A.27a)\ V0v)\ (ap\ (ap\ (c.2Ewords.2Eword_mul \\
& \quad A.27a)\ V0v)\ V1w)))))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0n \in ty.2Enum.2Enum.(\\
& \quad (ap\ (ap\ (c.2Ewords.2Eword_mul\ A.27a)\ (ap\ (c.2Ewords.2En2w\ A.27a) \\
V0n))\ (c.2Ewords.2Eword_L\ A.27a)) = (ap\ (ap\ (ap\ (c.2Ebool.2ECOND \\
& \quad (ty.2EfcP.2Ecart\ 2\ A.27a))\ (ap\ c.2Earithmetic.2EVEN\ V0n))\ (\\
& \quad ap\ (c.2Ewords.2En2w\ A.27a)\ c.2Enum.2E0))\ (c.2Ewords.2Eword_L \\
& \quad A.27a))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0n \in ty.2Enum.2Enum.(\\
& \quad (ap\ (ap\ (c.2Ewords.2Eword_mul\ A.27a)\ (ap\ (c.2Ewords.2Eword_2comp \\
& \quad A.27a)\ (ap\ (c.2Ewords.2En2w\ A.27a)\ V0n)))\ (c.2Ewords.2Eword_L \\
& \quad A.27a)) = (ap\ (ap\ (ap\ (c.2Ebool.2ECOND\ (ty.2EfcP.2Ecart\ 2\ A.27a)) \\
& \quad (ap\ c.2Earithmetic.2EVEN\ V0n))\ (ap\ (c.2Ewords.2En2w\ A.27a)\ c.2Enum.2E0)) \\
& \quad (c.2Ewords.2Eword_L\ A.27a))))
\end{aligned} \tag{35}$$

Theorem 1

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\ & nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow (((ap\ (ap\ (c_2Ewords_2Eword_mul \\ & A.27a)\ (c_2Ewords_2Eword_L2\ A.27a))\ (c_2Ewords_2Eword_L2 \\ & A.27a)) = (c_2Ewords_2Eword_L2\ A.27a)) \wedge (((ap\ (ap\ (c_2Ewords_2Eword_mul \\ & A.27b)\ (c_2Ewords_2Eword_L2\ A.27b))\ (c_2Ewords_2Eword_L2\ A.27b)) = \\ & (c_2Ewords_2Eword_L2\ A.27b)) \wedge ((\forall V0n \in ty_2Enum_2Enum. \\ & ((ap\ (ap\ (c_2Ewords_2Eword_mul\ A.27c)\ (ap\ (c_2Ewords_2En2w\ A.27c) \\ & V0n))\ (c_2Ewords_2Eword_L2\ A.27c)) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND \\ & (ty_2Efc_2Ecart\ 2\ A.27c))\ (ap\ c_2Earithmetic_2EEVEN\ V0n))\ (\\ & ap\ (c_2Ewords_2En2w\ A.27c)\ c_2Enum_2E0))\ (c_2Ewords_2Eword_L2 \\ & A.27c)))) \wedge (\forall V1n \in ty_2Enum_2Enum. ((ap\ (ap\ (c_2Ewords_2Eword_mul \\ & A.27d)\ (ap\ (c_2Ewords_2Eword_2comp\ A.27d)\ (ap\ (c_2Ewords_2En2w \\ & A.27d)\ V1n)))\ (c_2Ewords_2Eword_L2\ A.27d)) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND \\ & (ty_2Efc_2Ecart\ 2\ A.27d))\ (ap\ c_2Earithmetic_2EEVEN\ V1n))\ (\\ & ap\ (c_2Ewords_2En2w\ A.27d)\ c_2Enum_2E0))\ (c_2Ewords_2Eword_L2 \\ & A.27d)))))) \end{aligned}$$