thm_2Ewords_2Eword__abs (TMVmZ3odN5g4EL9W1GPKYp6jt1uqok4srxV)

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Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^{A_27$

Definition 5 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V2t \in 2$

Definition 6 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 7 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ $c_2Emin_2E_3D_3D_3E$\ V0t)$ c_2Ebool_2EF

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Efcp\_2Efinite\_\_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efcp\_2Efinite\_\_image\ A0) \tag{1}$$

Let $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ebool\_2Eitself\ A0) \tag{2}$$

Let $c\_2Ebool\_2Ethe\_\_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2Ethe\_\_value\ A\_27a \in (ty\_2Ebool\_2Eitself\ A\_27a) \tag{3}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{4}$$

Let $c\_2Efcp\_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Efcp\_2Edimindex\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (5)$$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \quad (6)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \quad (7)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (8)$$

Definition 9 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 10 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge$
of type $\iota \Rightarrow \iota$.

Definition 11 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Definition 12 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 13 *We define* c_2Ebool_2E_3F_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap$ c_2Ebool_2E_2F_5C

Definition 14 *We define* c_2Efcp_2Efinite__index *to be* $\lambda A_27a : \iota.(ap\ ($c_2Emin_2E_40 $(A\_27a^{ty\_2Enum\_2Enu}$

Let $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efcp\_2Ecart\ A0\ A1) \quad (9)$$

Let $c\_2Efcp\_2Edest\_\_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efcp\_2Edest\_\_cart\ A\_27a\ A\_27b \in ((A\_27a^{(ty\_2Efcp\_2Efinite\_\_image\ A\_27b)})^{(ty\_2Efcp\_2Ecart\ A\_27a\ A\_27b)}) \quad (10)$$

Definition 15 *We define* c_2Efcp_2Efcp__index *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in (ty\_2Efcp\_2Ecart\ A\_27$

Definition 16 *We define* c_2Efcp_2EFCP *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(ap\ ($

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \quad (11)$$

Definition 17 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 18 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{12}$$

Definition 19 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 20 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{13}$$

Definition 21 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 22 *We define* c_2Ebit_2ESBIT *to be* $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ ($c_2Eboo

Let $c\_2Esum\_\_num\_2ESUM : \iota$ be given. Assume the following.

$$c\_2Esum\_\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})^{ty\_2Enum\_2Enum}) \tag{14}$$

Definition 23 *We define* c_2Ewords_2Ew2n *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap\ (ap\ c$

Let $c\_2Ewords\_2Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2Edimword\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \tag{15}$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{16}$$

Definition 24 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let $c\_2Earithmetic\_2EDIV : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{17}$$

Definition 25 *We define* c_2Ebit_2EDIV__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Let $c\_2Earithmetic\_2EMOD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{18}$$

Definition 26 *We define* c_2Ebit_2EMOD__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2E$

Definition 27 *We define* c_2Ebit_2EBITS *to be* $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V$

Definition 28 *We define* c_2Ebit_2EBIT *to be* $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap$

Definition 29 *We define* c_2Ewords_2En2w *to be* $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.(ap$ (c_2Efcp_2EFC

Definition 30 *We define* c_2Ewords_2Eword__2comp *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).$

Definition 31 *We define* c_2Ewords_2Eword__msb *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap$

Let $ty\_2Epair\_2Eprod : \iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0\Rightarrow\forall A1.nonempty\ A1\Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{19}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{20}$$

Definition 32 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Definition 33 *We define* c_2Ebool_2ELET *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in (A\_27b^{A\_27a}).(\lambda V1x \in A\_27$

Definition 34 *We define* c_2Ewords_2Enzcv *to be* $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).\lambda V1b \in ($

Let $c\_2Epair\_2ESND : \iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{21}$$

Let $c\_2Epair\_2EFST : \iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{22}$$

Definition 35 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

Definition 36 *We define* c_2Ewords_2Eword__lt *to be* $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).\lambda V1b$

Definition 37 *We define* c_2Ewords_2Eword__abs *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap$

Assume the following.

$$True \tag{23}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p\ V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2)))))) \tag{24}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{25}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg (p\ V0t)))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{27}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{28}$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{29}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{30}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{32}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{33}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{34}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V0t1)\ V1t2) = V1t2)))) \tag{35}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{36}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{37}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in (ty\_2Efcp\_2Ecart\ A\_27a\ A\_27b).(\forall V1y \in (ty\_2Efcp\_2Ecart\ A\_27a\ A\_27b).((V0x = V1y) \Leftrightarrow (\forall V2i \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V2i)\ (ap\ (c\_2Efcp\_2Edimindex\ A\_27b)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27b)))) \Rightarrow ((ap\ (ap\ (\mathsf{c\_2Efcp\_2Efcp\_\_index}\ A\_27a\ A\_27b)\ V0x)\ V2i) = (ap\ (ap\ (\mathsf{c\_2Efcp\_2Efcp\_\_index}\ A\_27a\ A\_27b)\ V1y)\ V2i))))))) \tag{38}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(\forall V1i \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1i)\ (ap\ (c\_2Efcp\_2Edimindex\ A\_27b)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27b)))) \Rightarrow ((ap\ (ap\ (\mathsf{c\_2Efcp\_2Efcp\_\_index}\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Efcp\_2EFCP}\ A\_27a\ A\_27b)\ V0g))\ V1i) = (ap\ V0g\ V1i))))) \tag{39}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).((p\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_msb}\ A\_27a)\ V0w)) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_lt}\ A\_27a)\ V0w)\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ \mathsf{c\_2Enum\_2E0}))))) \tag{40}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).((ap\ (\mathsf{c\_2Ewords\_2Eword\_\_abs}\ A\_27a)\ V0w) = (ap\ (\mathsf{c\_2Efcp\_2EFCP}\ 2\ A\_27a)\ (\lambda V1i \in ty\_2Enum\_2Enum.(ap\ (ap\ \mathsf{c\_2Ebool\_2E\_5C\_2F}\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ \mathsf{c\_2Ebool\_2E\_7E}\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_msb}\ A\_27a)\ V0w)))\ (ap\ (ap\ (\mathsf{c\_2Efcp\_2Efcp\_\_index}\ 2\ A\_27a)\ V0w)\ V1i)))\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_msb}\ A\_27a)\ V0w))\ (ap\ (ap\ (\mathsf{c\_2Efcp\_2Efcp\_\_index}\ 2\ A\_27a)\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_2comp}\ A\_27a)\ V0w))\ V1i)))))))$$