thm_2Ewords_2Eword__concat__0__0 (TMQr-PrENukTQzuQNwnYzu1Yds1xSLMhStyD)

October 26, 2020

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{1}$$

Let $ty\_2Efcp\_2Efinite\_\_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efcp\_2Efinite\_\_image\ A0) \tag{2}$$

Definition 1 *We define* $\mathsf{c\_2Emin\_2E\_3D}$ *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Let $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ebool\_2Eitself\ A0) \tag{3}$$

Let $c\_2Ebool\_2Ethe\_\_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2Ethe\_\_value\ A\_27a \in (ty\_2Ebool\_2Eitself\ A\_27a) \tag{4}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{5}$$

Let $c\_2Efcp\_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Efcp\_2Edimindex\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \tag{6}$$

Definition 2 *We define* $\mathsf{c\_2Ebool\_2ET}$ *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* $\mathsf{c\_2Ebool\_2E\_21}$ *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* $\mathsf{c\_2Ebool\_2EF}$ *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2EF

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in 2$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{7}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{8}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{9}$$

Definition 8 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 10 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 11 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 12 *We define* c_2Ebool_2E_3F_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap$ c_2Ebool_2E_2F_5C

Definition 13 *We define* c_2Efcp_2Efinite__index *to be* $\lambda A\_27a : \iota.(ap$ (c_2Emin_2E_40 $(A\_27a^{ty\_2Enum\_2Enu}$

Let $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efcp\_2Ecart\ A0\ A1) \tag{10}$$

Let $c\_2Efcp\_2Edest\_\_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efcp\_2Edest\_\_cart\ A\_27a\ A\_27b \in ((A\_27a^{(ty\_2Efcp\_2Efinite\_\_image\ A\_27b)})^{(ty\_2Efcp\_2Ecart\ A\_27a\ A\_27b)}) \tag{11}$$

Definition 14 *We define* c_2Efcp_2Efcp__index *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in (ty\_2Efcp\_2Ecart\ A\_27$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{12}$$

Definition 15 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 16 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{13}$$

Definition 17 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 18 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{14}$$

Definition 19 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 20 *We define* c_2Ebit_2ESBIT *to be* $\lambda V0b \in 2.\lambda V1n \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap$ (c_2Eboo

Let $c\_2Esum\_\_num\_2ESUM : \iota$ be given. Assume the following.

$$c\_2Esum\_\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})^{ty\_2Enum\_2Enum}) \tag{15}$$

Definition 21 *We define* c_2Ewords_2Ew2n *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap\ (ap\ c$

Definition 22 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let $c\_2Earithmetic\_2EDIV : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{16}$$

Definition 23 *We define* c_2Ebit_2EDIV__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{17}$$

Let $c\_2Earithmetic\_2EMOD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{18}$$

Definition 24 *We define* c_2Ebit_2EMOD__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2E$

Definition 25 *We define* c_2Ebit_2EBITS *to be* $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V$

Definition 26 *We define* c_2Ebit_2EBIT *to be* $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap$

Definition 27 *We define* c_2Efcp_2EFCP *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(ap$

Definition 28 *We define* c_2Ewords_2En2w *to be* $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.(ap$ (c_2Efcp_2EFC

Definition 29 *We define* c_2Ewords_2Ew2w *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a$

Definition 30 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 31 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 32 *We define* c_2Ewords_2Eword__lsl *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).\lambda V1$

Definition 33 *We define* c_2Ewords_2Eword__or *to be* $\lambda A\_27a : \iota.\lambda V0v \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).\lambda V1v$

Definition 34 *We define* c_2Ebool_2ELET *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in (A\_27b^{A\_27a}).(\lambda V1x \in A\_27$

Definition 35 *We define* c_2Ewords_2Eword__join *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0v \in (ty\_2Efcp\_2Ecart\ 2$

Definition 36 *We define* c_2Ewords_2Eword__concat *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0v \in (ty\_2Ef$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((ap\ (\mathsf{c\_2Ewords\_2Ew2w}\ A\_27b\ A\_27a)\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27b)\ \mathsf{c\_2Enum\_2E0})) = (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ \mathsf{c\_2Enum\_2E0})) \quad (19)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0a \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).((ap\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_join}\ A\_27b\ A\_27a)\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27b)\ \mathsf{c\_2Enum\_2E0}))\ V0a) = (ap\ (\mathsf{c\_2Ewords\_2Ew2w}\ A\_27a\ (ty\_2Esum\_2Esum\ A\_27b\ A\_27a))\ V0a))) \quad (20)$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow ((ap\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_concat}\ A\_27a\ A\_27b\ A\_27c)\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ \mathsf{c\_2Enum\_2E0}))\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27b)\ \mathsf{c\_2Enum\_2E0})) = (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27c)\ \mathsf{c\_2Enum\_2E0}))$$