

thm\_2Ewords\_2Eword\_concat\_\_word\_\_T  
(TMWQPgW6bgkyuF9QmeyPZbuqMqztbEF6RP3)

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Let  $ty\_2Efc\_2Efinite\_image : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efc\_2Efinite\_image\ A0) \quad (1)$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$  of type  $\iota \Rightarrow \iota$ .

Let  $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ebool\_2Eitself\ A0) \quad (2)$$

Let  $c\_2Ebool\_2Ethe\_value : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2Ethe\_value\ A\_27a \in (ty\_2Ebool\_2Eitself\ A\_27a) \quad (3)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (4)$$

Let  $c\_2Efc\_2Edimindex : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Efc\_2Edimindex\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (5)$$

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27a})))$

**Definition 4** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2EF$

**Definition 7** We define  $c\_Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_Ebool\_2E\_21 2) (\lambda V2t \in 2)))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (6)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (7)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (8)$$

**Definition 8** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num (V0m))$

**Definition 9** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 10** We define  $c\_Ebool\_2E\_3F$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A) P)))$

**Definition 11** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap (c\_2Eprim\_rec\_2E\_3C V0m) V1n)$

**Definition 12** We define  $c\_Ebool\_2E\_3F\_21$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap c\_Ebool\_2E\_2F\_5C A) P))$

**Definition 13** We define  $c\_2Efcf\_2Efinite\_index$  to be  $\lambda A.27a : \iota.(ap (c\_2Emin\_2E\_40 (A\_27a)^{ty\_2Enum\_2Enum}) P)$

Let  $ty\_2Efcf\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Efcf\_2Ecart A0 A1) \quad (9)$$

Let  $c\_2Efcf\_2Edest\_cart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c\_2Efcf\_2Edest\_cart A.27a A.27b \in ((A.27a)^{ty\_2Efcf\_2Efinite\_image A.27b})^{ty\_2Efcf\_2Ecart A.27a A.27b} \quad (10)$$

**Definition 14** We define  $c\_2Efcf\_2Efcf\_index$  to be  $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0x \in (ty\_2Efcf\_2Ecart A.27a A.27b)$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (11)$$

**Definition 15** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

**Definition 16** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (12)$$

**Definition 17** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic$

**Definition 18** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Earithmetic\_2EEXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (13)$$

**Definition 19** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

**Definition 20** We define  $c\_2Ebit\_2ESBIT$  to be  $\lambda V0b \in 2.\lambda V1n \in ty\_2Enum\_2Enum.(ap (ap (ap (c\_2Ebo$

Let  $c\_2Esum\_num\_2ESUM : \iota$  be given. Assume the following.

$$c\_2Esum\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})^{ty\_2Enum\_2Enum}) \quad (14)$$

**Definition 21** We define  $c\_2Ewords\_2Ew2n$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2EfcP\_2Ecart\ 2\ A\_27a).(ap (ap$

**Definition 22** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic$

Let  $c\_2Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (15)$$

**Definition 23** We define  $c\_2Ebit\_2EDIV\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (16)$$

Let  $c\_2Earithmetic\_2EMOD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (17)$$

**Definition 24** We define  $c\_2Ebit\_2EMOD\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 25** We define  $c\_2Ebit\_2EBITS$  to be  $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V$

**Definition 26** We define  $c\_2Ebit\_2EBIT$  to be  $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap$

**Definition 27** We define  $c\_2EfcP\_2EFCP$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(ap$

**Definition 28** We define  $c\_2Ewords\_2En2w$  to be  $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.(ap (c\_2EfcP\_2EFC$

**Definition 29** We define  $c\_2Ewords\_2Ew2w$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0w \in (ty\_2EfcP\_2Ecart\ 2\ A\_27$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (18)$$

**Definition 30** We define  $c\_2Ebool\_2E5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E21\ 2)\ (\lambda V2t \in 2.$

**Definition 31** We define  $c\_2Earithmetic\_2E3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 32** We define  $c\_2Ewords\_2Eword\_2Isl$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).\lambda V1x \in 2.$

**Definition 33** We define  $c\_2Ewords\_2Eword\_2or$  to be  $\lambda A\_27a : \iota.\lambda V0v \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).\lambda V1x \in 2.$

**Definition 34** We define  $c\_2Ebool\_2ELET$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in (A\_27b^{A\_27a}).(\lambda V1x \in A\_27b.$

**Definition 35** We define  $c\_2Ewords\_2Eword\_2join$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0v \in (ty\_2Efc\_2Ecart\ 2\ A\_27a\ A\_27b).$

**Definition 36** We define  $c\_2Ewords\_2Eword\_2concat$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0v \in (ty\_2Efc\_2Ecart\ 2\ A\_27a\ A\_27b\ A\_27c).$

Let  $c\_2Ewords\_2Edimword : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2Edimword\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (19)$$

**Definition 37** We define  $c\_2Ewords\_2Eword\_22comp$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow & \\ (ap\ (ap\ (c\_2Ewords\_2Eword\_2join\ A\_27a\ A\_27b)\ (ap\ (c\_2Ewords\_2Eword\_22comp & \\ A\_27a)\ (ap\ (c\_2Ewords\_2En2w\ A\_27a)\ (ap\ c\_2Earithmetic\_2ENUMERAL & \\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))))))\ (ap\ (c\_2Ewords\_2Eword\_22comp & \\ A\_27b)\ (ap\ (c\_2Ewords\_2En2w\ A\_27b)\ (ap\ c\_2Earithmetic\_2ENUMERAL & \\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)))))) = (ap\ ( & \\ c\_2Ewords\_2Eword\_22comp\ (ty\_2Esum\_2Esum\ A\_27a\ A\_27b))\ (ap\ (c\_2Ewords\_2En2w & \\ (ty\_2Esum\_2Esum\ A\_27a\ A\_27b))\ (ap\ c\_2Earithmetic\_2ENUMERAL\ ( & \\ ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)))))) & \end{aligned} \quad (20)$$

**Theorem 1**

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. & \\ nonempty\ A\_27c \Rightarrow ((ap\ (ap\ (c\_2Ewords\_2Eword\_2concat\ A\_27a\ A\_27b & \\ A\_27c)\ (ap\ (c\_2Ewords\_2Eword\_22comp\ A\_27a)\ (ap\ (c\_2Ewords\_2En2w & \\ A\_27a)\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1 & \\ c\_2Earithmetic\_2EZERO))))))\ (ap\ (c\_2Ewords\_2Eword\_22comp\ A\_27b) & \\ (ap\ (c\_2Ewords\_2En2w\ A\_27b)\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap & \\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)))))) = (ap\ (c\_2Ewords\_2Ew2w & \\ (ty\_2Esum\_2Esum\ A\_27a\ A\_27b)\ A\_27c)\ (ap\ (c\_2Ewords\_2Eword\_22comp & \\ (ty\_2Esum\_2Esum\ A\_27a\ A\_27b))\ (ap\ (c\_2Ewords\_2En2w\ (ty\_2Esum\_2Esum & \\ A\_27a\ A\_27b))\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1 & \\ c\_2Earithmetic\_2EZERO)))))) & \end{aligned}$$