

thm_2Ewords_2Eword_dec_string
 (TMcRDzTSKroDgnzsiW1tNbnsuyHN8rQY29Q)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow p Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V1P \in 2.V1P)) (\lambda V2P \in 2.V2P)))$

Definition 5 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Definition 6 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g \in (A_27c^{A_27b}).\lambda V2h \in (A_27a^{A_27c}))$

Definition 7 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 8 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}).\lambda V1g \in ((A_27b^{A_27c})^{A_27a}).\lambda V2h \in ((A_27a^{A_27c})^{A_27b})))$

Definition 9 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A_27a})) A_27b (A_27b^{A_27a})))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (1)$$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (2)$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \quad (3)$$

Definition 10 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 11 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_21 2))$

Definition 12 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap(c_2Ebool_2E_21 2))(\lambda V2t \in$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (4)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (5)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (6)$$

Definition 13 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num$

Definition 14 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (\text{the } (\lambda x.x \in A \wedge$ of type $\iota \Rightarrow \iota$.

Definition 15 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap(c_2Emin_2E_40$

Definition 16 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 17 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 18 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (7)$$

Definition 19 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Definition 20 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 21 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap (ap (ap (c_2Ebool_2E$

Let $c_2Earithmetic_2EEEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (8)$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (9)$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (10)$$

Definition 22 We define $c_2Enumeral_2EiiSUC$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap c_2Enum_2ESUC (ap$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (11)$$

Definition 23 We define $c_2Eenumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Definition 24 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. V1n$.

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Ebool_2EARB A_27a \in A_27a \quad (12)$$

Definition 25 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Definition 26 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2Ebool) n)$

Definition 27 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Definition 28 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2Ebool) n)$

Let $ty_2Estring_2Echar : \iota$ be given. Assume the following.

$$nonempty ty_2Estring_2Echar \quad (13)$$

Let $c_2Estring_2ECHR : \iota$ be given. Assume the following.

$$c_2Estring_2ECHR \in (ty_2Estring_2Echar^{ty_2Enum_2Enum}) \quad (14)$$

Definition 29 We define $c_2Ebool_2Eliteral_case$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0f \in (A_27b^{A_27a}). (\lambda V1x$

Definition 30 We define $c_2Erelation_2EWF$ to be $\lambda A_27a : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). (ap (c_2Ebool_2E_21$

Definition 31 We define $c_2Erelation_2ERESTRIC$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1l$

Definition 32 We define $c_2Erelation_2ETC$ to be $\lambda A_27a : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1a \in A_27a. \lambda V2b$

Definition 33 We define $c_2Erelation_2Eapprox$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1M$

Definition 34 We define $c_2Erelation_2Ethe_fun$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1M$

Definition 35 We define $c_2Erelation_2EWFREC$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1M$

Definition 36 We define $c_2EASCIInumbers_2EUNHEX$ to be $(ap (ap (c_2Erelation_2EWFREC ty_2Estring$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_0. \text{nonempty } A_0 \Rightarrow \text{nonempty } (\text{ty_2Elist_2Elist } A_0) \quad (15)$$

Let $c_2Elist_2EREVERSE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2Elist_2EREVERSE\ A_27a \in ((ty_2Elist_2Elist\\ A_27a)^{(ty_2Elist_2Elist\ A_27a)}) \quad (16)$$

Let $c_2Elist_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.\text{nonempty } A.27a \Rightarrow \forall A.27b.\text{nonempty } A.27b \Rightarrow c.2Elist.2EMAP \\ A.27a \ A.27b \in (((ty.2Elist.2Elist A.27b)^{(ty.2Elist.2Elist A.27a)})^{(A.27b^A)^{27a}}) \quad (17)$$

Let $c_2Enumposrep_2El2n : \iota$ be given. Assume the following.

$$c_2Enumposrep_2El2n \in ((ty_2Enum_2Enum^{(ty_2Elist_2Elist\ ty_2Enum_2Enum)})^{ty_2Enum_2Enum}) \quad (18)$$

Definition 37 We define $c_<\!\!2EASCIInumbers_<\!\!2Es2n$ to be $\lambda V0b \in ty_2Enum_2Enum. \lambda V1f \in (ty_2Enum_2Enum \rightarrow ty_2Enum_2Enum) .$

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (19)$$

Definition 38 We define $c_2EBit_2EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Let c_2 be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (20)$$

Definition 39 We define `c_2EBit_2EMOD_2EXP` to be $\lambda V0x \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Definition 40 We define c_2EBit_2EBITS to be $\lambda V0h \in ty_2Enum_2Enum. \lambda V1l \in ty_2Enum_2Enum. \lambda V2m \in ty_2Enum_2Enum. \lambda V3n \in ty_2Enum_2Enum.$

Definition 41 We define `c_2EBit_2EBIT` to be $\lambda V0b \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.(ap$

Let $ty_2Efc_{\text{cp}}_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty_}2E\text{fc}\text{p_}2E\text{finite_image } A0) \quad (21)$$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty_2Ebool_2Eitself } A0) \quad (22)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2Ebool_2Ethe_value \ A_27a \in (ty_2Ebool_2Eitself \ A_27a) \quad (23)$$

Let $c_2Efcp_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2Efcp_2Edimindex A_27a \in (\text{ty_2Enum_2Enum}^{(\text{ty_2Ebool_2Eitself } A_27a)})$$

(24)

Definition 42 We define $c_2Ebool_2E_3F_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^A \rightarrow 2^a)).(ap\ (ap\ c_2Ebool_2E_2F_5G\ P\ V)\ 0)$

Definition 43 We define $c_2Efcp_2Efinite_index$ to be $\lambda A_27a : \iota.(ap\ (c_2Emin_2E40\ (A_27a))$

Let $ty_2Efcp_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow \forall A1.\text{nonempty } A1 \Rightarrow \text{nonempty}(\text{ty_2Efcp_2Ecart } A0\ A1) \quad (25)$$

Let $c_2Efcp_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow c_2Efc_2Edest_cart A_27a A_27b \in ((A_27a^{(ty_2Efc_2Efinite_image A_27b)})^{(ty_2Efc_2Ecart A_27a A_27b)}) \quad (26)$$

Definition 44 We define $c_2Efcp_2Efcp_index$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0x \in (ty_2Efcp_2Ecart\ A.27)$

Definition 45 We define c_2Efcp_2EFCP to be $\lambda A._27a : \iota.\lambda A._27b : \iota.(\lambda V0g \in (A._27a^{ty_2Enum_2Enum}).(ap$

Definition 46 We define $c_2Ewords_2En2w$ to be $\lambda A_27a : \iota.\lambda V^{\alpha}n \in ty_2Enum_2Enum.(ap\ (c_2Efcp_2EFC\ A\ V\ n)\ (c_2Ewords_2En2w\ A\ V\ n))$

Definition 48 We define $c_2Ewords_2Eword_from_dec_string$ to be $\lambda A__27a : \iota.(ap\ (ap\ (c_2Ewords_2Es2w\ A__27a)\ (dec_string\ A__27a))\ (c_2Ewords_2Eword\ A__27a))$

Definition 49 We define c_2EASCIInumbers_2EHEX to be (ap (ap (c_2Erelation_2EWFREC ty_2Enum_2E

Definition 50 We define c_2Ebit_2ESBIT to be $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ (ap\ (c_2Ebit\$

Let $c_2Esum_num_2ESUM : \iota$ be given. Assume the following.

$$c_2Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})^{ty_2Enum_2Enum}) \quad (27)$$

Definition 51 We define $c_2Ewords_2Ew2n$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efcp_2Ecart\ 2\ A_27a).(ap\ (ap\ c\ w)\ V0)$

Let $c_2Enumposrep_2En2l : \iota$ be given. Assume the following.

$$c_2Enumposrep_2En2l \in (((ty_2Elist_2Elist\ ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (28)$$

Definition 52 We define $c_2EASCIInumbers_2En2s$ to be $\lambda V0b \in ty_2Enum_2Enum. \lambda V1f \in (ty_2Estring$

Definition 53 We define $c_2Ewords_2Ew2s$ to be $\lambda A_27a : \iota.\lambda V0b \in ty_2Enum_2Enum.\lambda V1f \in (ty_2Estr,$

Definition 54 We define $c_2Ewords_2Eword_to_dec_string$ to be $\lambda A_\mathit{27a} : \iota.(ap\ (ap\ (c_2Ewords_2Ew2s\ A_\mathit{27a})\ (c_2Ewords_2Ew1s\ A_\mathit{27a}))\ (\lambda x\ x))$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C V0n) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 \\
 & (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 \\
 & c_2Earithmetic_2EZERO))))))) \Rightarrow ((ap c_2EASCIInumbers_2EUNHEX \\
 & (ap c_2EASCIInumbers_2EHEX V0n)) = V0n)))
 \end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
 & (p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
 & (ap c_2Enum_2ESUC V0m)) V1n)))) \\
 & (ap c_2Enum_2E0) V0n)))
 \end{aligned} \tag{30}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (p (ap (ap c_2Earithmetic_2E_3C_3D \\
 c_2Enum_2E0) V0n))) \tag{31}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
 & (\neg(p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
 & V1n) V0m)))) \\
 & (ap c_2Enum_2E0) V0n)))
 \end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
 & ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge \\
 & (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge \\
 & (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL \\
 & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge \\
 & (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL \\
 & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge \\
 & ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap \\
 & (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) \\
 & V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = \\
 & (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A \\
 & V0m) V1n)))))))
 \end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
 & \forall V2p \in ty_2Enum_2Enum. (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
 & V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))) \Rightarrow (p (\\
 & ap (ap c_2Earithmetic_2E_3C_3D V0m) V2p))))))
 \end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
 & \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D \\
 & (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) (ap (ap c_2Earithmetic_2E_2B \\
 & V0m) V2p))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p)))))) \\
 \end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC V0n) = (ap (ap \\
 & c_2Earithmetic_2E_2B (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
 & c_2Earithmetic_2EZERO)) V0n))) \\
 \end{aligned} \tag{36}$$

Assume the following.

$$True \tag{37}$$

Assume the following.

$$\begin{aligned}
 & \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in \\
 & A_27a. (p V0t)) \Leftrightarrow (p V0t))) \\
 \end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\
 & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\
 & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \\
 \end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
 & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge \\
 & (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \\
 \end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
 & ((\forall V0t \in 2. ((\neg(\neg(p V0t)) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\
 & ((\neg False) \Leftrightarrow True))) \\
 \end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
 & \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow \\
 & True)) \\
 \end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
 & \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\
 & A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \\
 \end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
 & \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow \\
 & \forall V0f \in (A_27b^{A_27a}). (\forall V1g \in (A_27b^{A_27a}). ((V0f = \\
 & V1g) \Leftrightarrow (\forall V2x \in A_27a. ((ap V0f V2x) = (ap V1g V2x))))) \\
 \end{aligned} \tag{44}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (45)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee (p V1B))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge (\neg(p V1B))))))) \quad (46)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))) \quad (47)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Leftrightarrow ((p V0t) \Leftrightarrow False))) \quad (48)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (49)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in 2.(((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27))))))) \quad (50)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow \forall A_27c. \\ & \quad \text{nonempty } A_27c \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27a^{A_27c}). \\ & \quad (\forall V2x \in A_27c.((ap (ap (ap (c_2Ecombin_2Eo A_27c A_27b A_27a) \\ & \quad V0f) V1g) V2x) = (ap V0f (ap V1g V2x))))))) \end{aligned} \quad (51)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.((ap (c_2Ecombin_2E1 \\ & \quad A_27a) V0x) = V0x)) \quad (52)$$

Assume the following.

$$\begin{aligned} & (((ap c_2Enum_2ESUC c_2Earithmetic_2EZERO) = (ap c_2Earithmetic_2EBIT1 \\ & \quad c_2Earithmetic_2EZERO)) \wedge ((\forall V0n \in ty_2Enum_2Enum.((ap \\ & \quad c_2Enum_2ESUC (ap c_2Earithmetic_2EBIT1 V0n)) = (ap c_2Earithmetic_2EBIT2 \\ & \quad V0n))) \wedge (\forall V1n \in ty_2Enum_2Enum.((ap c_2Enum_2ESUC (ap c_2Earithmetic_2EBIT2 \\ & \quad V1n)) = (ap c_2Earithmetic_2EBIT1 (ap c_2Enum_2ESUC V1n))))))) \end{aligned} \quad (53)$$

Assume the following.

$((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in ty_2Enum_2Enum.((\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enum_2EiZ (ap (ap c_2Earithmetic_2E_2B V2n) V3m))))))) \wedge ((\forall V4n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum.((\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A V6n) V7m))))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in ty_2Enum_2Enum.((\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V10n) V11m))))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEEXP V14n) c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge ((\forall V15n \in ty_2Enum_2Enum.((\forall V16m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEEXP (ap c_2Earithmetic_2ENUMERAL V15n)) (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2EEEXP V15n) V16m))))))) \wedge (((ap c_2Enum_2ESUC c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \wedge ((\forall V17n \in ty_2Enum_2Enum.((ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enum_2ESUC V17n)))))) \wedge (((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Eprim_rec_2EPRE V18n)))))) \wedge ((\forall V19n \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum.((\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))))) \wedge ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL V23n)) \Leftrightarrow False))) \wedge ((\forall V24n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL V24n)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) V24n)))) \wedge ((\forall V25n \in ty_2Enum_2Enum.((\forall V26m \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL V25n)) (ap c_2Earithmetic_2ENUMERAL V26m)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL V25n) V26m))))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E c_2Enum_2E0) V27n)) \Leftrightarrow False))) \wedge ((\forall V28n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL V28n)) c_2Enum_2E0) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) V28n)))) \wedge ((\forall V29n \in ty_2Enum_2Enum.((\forall V30m \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E c_2Enum_2E0) V29n)) (ap c_2Earithmetic_2ENUMERAL V30m)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V29n))))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3D c_2Enum_2E0) V31n)) \Leftrightarrow True))) \wedge ((\forall V32n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C c_2Enum_2E0) V32n)) \Leftrightarrow False))) \wedge ((p (ap (ap c_2Earithmetic_2E_3D c_2Enum_2E0) V32n)) \Leftrightarrow True))) \wedge ((\forall V33n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3D c_2Enum_2E0) V33n)) \Leftrightarrow False))) \wedge ((p (ap (ap c_2Earithmetic_2E_3D c_2Enum_2E0) V33n)) \Leftrightarrow True)))$

Assume the following.

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) (ap c_2Earithmetic_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& (ap c_2Earithmetic_2EBIT2 V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& V0n) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT1 V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT2 V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT1 V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow \\
& (\neg(p (ap (ap c_2Eprim_rec_2E_3C V1m) V0n))) \wedge ((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT2 V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))))))))))) \\
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Earithmetic_2E_3C_3D c_2Earithmetic_2EZERO) V0n))) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap c_2Earithmetic_2EBIT2 V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (\neg(p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V1m) V0n))) \wedge ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))))))))))) \\
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}. nonempty\ A_{.27a} \Rightarrow (\forall V0c2n \in (ty_2Enum_2Enum^{ty_2Estring_2Echar}). \\
& (\forall V1n2c \in (ty_2Estring_2Echar^{ty_2Enum_2Enum}). (\forall V2b \in \\
& ty_2Enum_2Enum. (\forall V3w \in (ty_2Efcpc_2Ecart\ 2\ A_{.27a}). (((\\
& p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL (ap \\
& c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V2b)) \wedge (\forall V4x \in \\
& ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C V4x) V2b)) \Rightarrow ((\\
& ap V0c2n (ap V1n2c V4x)) = V4x))) \Rightarrow ((ap (ap (c_2Ewords_2Es2w \\
& A_{.27a}) V2b) V0c2n) (ap (ap (c_2Ewords_2Es2w A_{.27a}) V2b) V1n2c) \\
& V3w)) = V3w))))))) \\
\end{aligned} \tag{58}$$

Theorem 1

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow ((\text{ap} (\text{ap} (\text{c_Ecombin_2Eo} (\text{ty_2Efcp_2Ecart} \\ 2 A_27a) (\text{ty_2Efcp_2Ecart} 2 A_27a) (\text{ty_2Elist_2Elist} \text{ty_2Estring_2Echar})) \\ (\text{c_Ewords_2Eword_from_dec_string} A_27a)) (\text{c_Ewords_2Eword_to_dec_string} \\ A_27a)) = (\text{c_Ecombin_2EI} (\text{ty_2Efcp_2Ecart} 2 A_27a)))$$