

thm_2Ewords_2Eword__ge__n2w
(TMczxz5i3NFq6n5kWAwdBJomk9fk6JrGS9j)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ELET to be $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.(\lambda V0f \in (A.\lambda 27b^{A.\lambda 27a}).(\lambda V1x \in A.\lambda 27a$

Definition 3 We define c_2Ebool_2EET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V$

Definition 4 We define $c_2Ecombin_2EK$ to be $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.(\lambda V0x \in A.\lambda 27a.(\lambda V1y \in A.\lambda 27b.V0x))$

Definition 5 We define $c_2Ecombin_2ES$ to be $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.\lambda A.\lambda c : \iota.(\lambda V0f \in ((A.\lambda 27c^{A.\lambda 27b})^{A.\lambda 27a}$

Definition 6 We define $c_2Ecombin_2EI$ to be $\lambda A.\lambda a : \iota.(ap (ap (c_2Ecombin_2ES A.\lambda 27a (A.\lambda 27a^{A.\lambda 27a}) A.$

Definition 7 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A.\lambda 27a}).(ap (ap (c_2Emin_2E_3D (2^{A.\lambda 27a}$

Definition 8 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 9 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow p Q)$ of type ι .

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 10 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (4)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (5)$$

Definition 11 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 12 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ebool_2Eitself\ A0) \quad (8)$$

Let $c_2Ewords_2Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2Edimword\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (9)$$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (10)$$

Definition 13 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Definition 14 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 15 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2Ethe_value\ A_27a \in (ty_2Ebool_2Eitself\ A_27a) \quad (11)$$

Let $c_2EfcP_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2EfcP_2Edimindex\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (12)$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (13)$$

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (14)$$

Definition 16 We define $c_Ebit_2EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 17 We define $c_Ebit_2EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 18 We define c_Ebit_2EBITS to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.\lambda V$

Definition 19 We define c_Ebit_2EBIT to be $\lambda V0b \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap$

Let $ty_2Efc_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Efc_2Efinite_image\ A0) \quad (15)$$

Definition 20 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E$

Definition 21 We define $c_Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 22 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A \wedge$
of type $\iota \Rightarrow \iota$.

Definition 23 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 24 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 25 We define $c_Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ c_2Ebool_2E_2F_5C$

Definition 26 We define $c_2Efc_2Efinite_index$ to be $\lambda A_27a : \iota.(ap\ (c_2Emin_2E_40\ (A_27a^{ty_2Enum_2Enum}$

Let $ty_2Efc_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Efc_2Ecart\ A0\ A1) \quad (16)$$

Let $c_2Efc_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efc_2Edest_cart\ A_27a\ A_27b \in ((A_27a^{(ty_2Efc_2Efinite_image\ A_27b)})^{(ty_2Efc_2Ecart\ A_27a\ A_27b)}) \quad (17)$$

Definition 27 We define $c_2Efc_2Efc_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efc_2Ecart\ A_27a\ A_27b)$

Definition 28 We define c_2Efc_2EFCP to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap\ (ap\ (ap\ (c_2Emin_2E_40$

Definition 29 We define $c_2Ewords_2En2w$ to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.(ap\ (c_2Efc_2EFCP\ A_27a\ n)$

Definition 30 We define c_Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap\ (ap\ (ap\ (c_2Emin_2E_40$

Definition 31 We define c_Ebit_2ESBIT to be $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ (c_2Emin_2E_40$

Let $c_2Esum_num_2ESUM : \iota$ be given. Assume the following.

$$c_2Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})^{ty_2Enum_2Enum}) \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((ap\ (c_2Ecombin_2El\ A_27a)\ V0x) = V0x)) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (29)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (32)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow ((p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \vee (\neg(p\ V1q))) \wedge (((p\ V1q) \vee (\neg(p\ V2r))) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \quad (33)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow ((p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \quad (34)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a. \text{nonempty } A_27a \Rightarrow ((\text{ap } (c_2Ewords_2Edimword } A_27a) \\
& (c_2Ebool_2Ethe_value } A_27a)) = (\text{ap } (\text{ap } c_2Earithmetic_2EEXP \\
& (\text{ap } c_2Earithmetic_2ENUMERAL } (\text{ap } c_2Earithmetic_2EBIT2 } c_2Earithmetic_2EZERO))) \\
& (\text{ap } (c_2Efcf_2Edimindex } A_27a) (c_2Ebool_2Ethe_value } A_27a)))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0n \in \text{ty_2Enum_2Enum.} (\\
& (\text{ap } (c_2Ewords_2Ew2n } A_27a) (\text{ap } (c_2Ewords_2En2w } A_27a) V0n)) = \\
& (\text{ap } (\text{ap } c_2Earithmetic_2EMOD } V0n) (\text{ap } (c_2Ewords_2Edimword } A_27a) \\
& (c_2Ebool_2Ethe_value } A_27a))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0n \in \text{ty_2Enum_2Enum.} (\\
& (p (\text{ap } (c_2Ewords_2Eword_msb } A_27a) (\text{ap } (c_2Ewords_2En2w } A_27a) \\
& V0n))) \Leftrightarrow (p (\text{ap } (\text{ap } c_2Ebit_2EBIT } (\text{ap } (\text{ap } c_2Earithmetic_2E_2D } (\\
& \text{ap } (c_2Efcf_2Edimindex } A_27a) (c_2Ebool_2Ethe_value } A_27a))) \\
& (\text{ap } c_2Earithmetic_2ENUMERAL } (\text{ap } c_2Earithmetic_2EBIT1 } c_2Earithmetic_2EZERO)))) \\
& V0n)))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0a \in (\text{ty_2Efcf_2Ecart} \\
& 2 } A_27a). (\forall V1b \in (\text{ty_2Efcf_2Ecart } 2 } A_27a). ((p (\text{ap } (\text{ap } \\
& (c_2Ewords_2Eword_ge } A_27a) V0a) V1b)) \Leftrightarrow (((p (\text{ap } (c_2Ewords_2Eword_msb \\
& A_27a) V1b)) \Leftrightarrow (p (\text{ap } (c_2Ewords_2Eword_msb } A_27a) V0a))) \wedge (p (\\
& \text{ap } (\text{ap } c_2Earithmetic_2E_3E_3D } (\text{ap } (c_2Ewords_2Ew2n } A_27a) V0a)) \\
& (\text{ap } (c_2Ewords_2Ew2n } A_27a) V1b)))) \vee ((p (\text{ap } (c_2Ewords_2Eword_msb \\
& A_27a) V1b)) \wedge (\neg(p (\text{ap } (c_2Ewords_2Eword_msb } A_27a) V0a))))))
\end{aligned} \tag{40}$$

Theorem 1

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0a \in ty_2Enum_2Enum.(\\ & \quad \forall V1b \in ty_2Enum_2Enum.((p\ (ap\ (ap\ (c_2Ewords_2Eword_ge \\ & \quad A_{.27a})\ (ap\ (c_2Ewords_2En2w\ A_{.27a})\ V0a))\ (ap\ (c_2Ewords_2En2w \\ & \quad A_{.27a})\ V1b))) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2ELET\ 2\ 2)\ (ap\ (ap\ (c_2Ebool_2ELET \\ & \quad 2\ (2^2))\ (\lambda V2sa \in 2.(\lambda V3sb \in 2.(ap\ (ap\ c_2Ebool_2E_5C_2F \\ & \quad (ap\ (ap\ c_2Ebool_2E_2F_5C\ (ap\ (ap\ (c_2Emin_2E_3D\ 2)\ V2sa)\ V3sb)) \\ & \quad (ap\ (ap\ c_2Earithmetic_2E_3E_3D\ (ap\ (ap\ c_2Earithmetic_2EMOD \\ & \quad V0a)\ (ap\ (c_2Ewords_2Edimword\ A_{.27a})\ (c_2Ebool_2Ethe_value \\ & \quad A_{.27a))))\ (ap\ (ap\ c_2Earithmetic_2EMOD\ V1b)\ (ap\ (c_2Ewords_2Edimword \\ & \quad A_{.27a})\ (c_2Ebool_2Ethe_value\ A_{.27a))))))\ (ap\ (ap\ c_2Ebool_2E_2F_5C \\ & \quad (ap\ c_2Ebool_2E_7E\ V2sa))\ V3sb))))\ (ap\ (ap\ c_2Ebit_2EBIT\ (ap\ (\\ & \quad ap\ c_2Earithmetic_2E_2D\ (ap\ (c_2Efc_2Edimindex\ A_{.27a})\ (c_2Ebool_2Ethe_value \\ & \quad A_{.27a)))\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1 \\ & \quad c_2Earithmetic_2EZERO))))\ V0a)))\ (ap\ (ap\ c_2Ebit_2EBIT\ (ap\ (ap \\ & \quad c_2Earithmetic_2E_2D\ (ap\ (c_2Efc_2Edimindex\ A_{.27a})\ (c_2Ebool_2Ethe_value \\ & \quad A_{.27a)))\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1 \\ & \quad c_2Earithmetic_2EZERO))))\ V1b)))))) \end{aligned}$$