

# thm\_2Ewords\_2Eword\_\_join\_\_index (TMUp- PcBUowegkPjfSR44LMYjdu8iGCAXYsG)

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**Definition 1** We define  $c\_2Emin\_2E3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

**Definition 3** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}))$

**Definition 4** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A\_27a : \iota.(ap (ap (c\_2Ecombin\_2ES A\_27a (A\_27a^{A\_27a})) A\_27a))$

**Definition 5** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 6** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2ET)$ .

**Definition 7** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}). (ap V1f V0x)))$

**Definition 8** We define  $c\_2Emin\_2E3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 9** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}). (ap (ap (c\_2Emin\_2E3D (2^{A\_27a}))$

**Definition 10** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

**Definition 11** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (1)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (2)$$



**Definition 22** We define  $c\_Eprim\_rec\_E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 23** We define  $c\_Earithmetic\_E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 24** We define  $c\_Earithmetic\_E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum.$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{10}$$

**Definition 25** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP).$

**Definition 26** We define  $c\_Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ (ap\ (ap\ (c\_2Ebool\_2E2E$

Let  $c\_2Earithmetic\_2EEXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{11}$$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{12}$$

**Definition 27** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x.$

**Definition 28** We define  $c\_2Enumeral\_2EiSUC$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap\ c\_2Enum\_2ESUC\ (ap$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{13}$$

**Definition 29** We define  $c\_2Enumeral\_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x.$

**Definition 30** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap\ (ap\ c\_2Earithmetic\_2E\_2B$

**Definition 31** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap\ (ap\ c\_2Earithmetic\_2E\_2B$

**Definition 32** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0.$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{14}$$

**Definition 33** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum.$

Let  $ty\_2Efc\_2Efinite\_image : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efc\_2Efinite\_image\ A0) \quad (15)$$

Let  $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ebool\_2Eitself\ A0) \quad (16)$$

Let  $c\_2Ebool\_2Ethe\_value : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Ebool\_2Ethe\_value\ A\_27a \in (ty\_2Ebool\_2Eitself\ A\_27a) \quad (17)$$

Let  $c\_2Efc\_2Edimindex : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Efc\_2Edimindex\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (18)$$

**Definition 34** We define  $c\_2Ebool\_2E\_3F\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ (ap\ c\_2Ebool\_2E\_2F\_20\ P)))$

**Definition 35** We define  $c\_2Efc\_2Efinite\_index$  to be  $\lambda A\_27a : \iota. (ap\ (c\_2Emin\_2E\_40\ (A\_27a^{ty\_2Enum\_2Enum})))$

Let  $ty\_2Efc\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efc\_2Ecart\ A0\ A1) \quad (19)$$

Let  $c\_2Efc\_2Edest\_cart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow c\_2Efc\_2Edest\_cart\ A\_27a\ A\_27b \in ((A\_27a^{(ty\_2Efc\_2Efinite\_image\ A\_27b)})^{(ty\_2Efc\_2Ecart\ A\_27a\ A\_27b)}) \quad (20)$$

**Definition 36** We define  $c\_2Efc\_2Efc\_index$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in (ty\_2Efc\_2Ecart\ A\_27a\ A\_27b)$

**Definition 37** We define  $c\_2Efc\_2EFCP$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}). (ap\ (ap\ c\_2Efc\_2EFCP\ g)))$

**Definition 38** We define  $c\_2Ewords\_2Eword\_lsl$  to be  $\lambda A\_27a : \iota. \lambda V0w \in (ty\_2Efc\_2Ecart\ 2\ A\_27a). \lambda V1x \in (ty\_2Efc\_2Ecart\ A\_27a\ A\_27w)$

**Definition 39** We define  $c\_2Ewords\_2Eword\_or$  to be  $\lambda A\_27a : \iota. \lambda V0v \in (ty\_2Efc\_2Ecart\ 2\ A\_27a). \lambda V1x \in (ty\_2Efc\_2Ecart\ A\_27a\ A\_27v)$

**Definition 40** We define  $c\_2Ebool\_2ELET$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0f \in (A\_27b^{A\_27a}). (\lambda V1x \in A\_27b. (c\_2Ebool\_2Ethe\_value\ x)))$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (21)$$

**Definition 41** We define  $c\_2Ebit\_2ESBIT$  to be  $\lambda V0b \in 2. \lambda V1n \in ty\_2Enum\_2Enum. (ap\ (ap\ (ap\ (c\_2Ebool\_2Ethe\_value\ b))))$

Let  $c\_2Esum\_num\_2ESUM : \iota$  be given. Assume the following.

$$c\_2Esum\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}})^{ty\_2Enum\_2Enum}) \quad (22)$$

**Definition 42** We define  $c\_2Ewords\_2Ew2n$  to be  $\lambda A\_27a : \iota. \lambda V0w \in (ty\_2Efc\_2Ecart\ 2\ A\_27a). (ap\ (ap\ c\_2Esum\_num\_2ESUM\ V0w)\ V1n)$

Let  $c\_2Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (23)$$

**Definition 43** We define  $c\_2Ebit\_2EDIV\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum. (ap\ (ap\ c\_2Earithmetic\_2EDIV\ V0x)\ V1n)$

Let  $c\_2Earithmetic\_2EMOD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (24)$$

**Definition 44** We define  $c\_2Ebit\_2EMOD\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum. (ap\ (ap\ c\_2Earithmetic\_2EMOD\ V0x)\ V1n)$

**Definition 45** We define  $c\_2Ebit\_2EBITS$  to be  $\lambda V0h \in ty\_2Enum\_2Enum. \lambda V1l \in ty\_2Enum\_2Enum. \lambda V2n \in ty\_2Enum\_2Enum. (ap\ (ap\ (ap\ c\_2Ebit\_2EBIT\ V0h)\ V1l)\ V2n)$

**Definition 46** We define  $c\_2Ebit\_2EBIT$  to be  $\lambda V0b \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum. (ap\ (ap\ c\_2Ebit\_2EBIT\ V0b)\ V1n)$

**Definition 47** We define  $c\_2Ewords\_2En2w$  to be  $\lambda A\_27a : \iota. \lambda V0n \in ty\_2Enum\_2Enum. (ap\ (c\_2Efc\_2Ecart\ 2\ A\_27a)\ V0n)$

**Definition 48** We define  $c\_2Ewords\_2Ew2w$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0w \in (ty\_2Efc\_2Ecart\ 2\ A\_27a). (ap\ (ap\ (ap\ c\_2Efc\_2Ecart\ 2\ A\_27b)\ V0w)\ V1n)$

**Definition 49** We define  $c\_2Ewords\_2Eword\_join$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0v \in (ty\_2Efc\_2Ecart\ 2\ A\_27a). (ap\ (ap\ (ap\ c\_2Efc\_2Ecart\ 2\ A\_27b)\ V0v)\ V1n)$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ & ((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Enum\_2E0)\ V0m) = V0m) \wedge (((ap\ ( \\ & ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ c\_2Enum\_2E0) = V0m) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2B \\ & (ap\ c\_2Enum\_2ESUC\ V0m))\ V1n) = (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\ & V0m)\ V1n))) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ c\_2Enum\_2ESUC \\ & V1n)) = (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ & (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\ & V1n)\ V0m)))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ & (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\ & V1n)\ V0m)))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad \forall V2p \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2B V0m) \\
& (ap (ap c\_2Earithmetic\_2E\_2B V1n) V2p)) = (ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) V2p))))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad (ap c\_2Enum\_2ESUC V0m)) V1n))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad c\_2Enum\_2E0) V0n)))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& (\neg (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad V1n) V0m))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2A ( \\
& ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) \\
& \quad V0m) = V0m))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V0m) = c\_2Enum\_2E0) \wedge \\
& (((ap (ap c\_2Earithmetic\_2E\_2A V0m) c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge \\
& \quad (((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0m) = V0m) \wedge \\
& \quad (((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = V0m) \wedge ( \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)) \\
& \quad V1n)) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Enum\_2ESUC V1n)) = \\
& \quad (ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V0m) V1n))))))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad \forall V2p \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2A (ap \\
& (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) V2p) = (ap (ap c\_2Earithmetic\_2E\_2B \\
& (ap (ap c\_2Earithmetic\_2E\_2A V0m) V2p)) (ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V1n) V2p)))))) \\
& \hspace{15em} (34)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad \forall V2p \in ty\_2Enum\_2Enum. (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0m) V1n)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p))) \Rightarrow (p ( \\
& \quad ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V2p)))))) \\
& \hspace{15em} (35)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad \forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) (ap (ap c\_2Earithmetic\_2E\_2B \\
& V0m) V2p))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p)))))) \\
& \hspace{15em} (36)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad (\neg (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad (ap c\_2Enum\_2ESUC V1n)) V0m)))))) \\
& \hspace{15em} (37)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad (\neg (V0m = V1n)) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC \\
& V0m)) V1n)) \vee (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC \\
& \quad V1n)) V0m)))))) \\
& \hspace{15em} (38)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. ((ap c\_2Enum\_2ESUC V0n) = (ap (ap \\
& c\_2Earithmetic\_2E\_2B (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad c\_2Earithmetic\_2EZERO))) V0n))) \\
& \hspace{15em} (39)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad \forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Eprim\_rec\_2E\_3C ( \\
& ap (ap c\_2Earithmetic\_2E\_2D V0m) V1n)) V2p)) \Leftrightarrow ((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& V0m) (ap (ap c\_2Earithmetic\_2E\_2B V1n) V2p))) \wedge (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad c\_2Enum\_2E0) V2p)))))) \\
& \hspace{15em} (40)
\end{aligned}$$

Assume the following.

$$True \quad (41)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2)))))) \quad (42)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p \ V0t))) \quad (43)$$

Assume the following.

$$(\forall V0t \in 2.((p \ V0t) \vee (\neg(p \ V0t)))) \quad (44)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow \forall A\_27b.nonempty \ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1x \in A\_27a.((ap \ (ap \ (c\_2Ebool\_2ELET \ A\_27a \ A\_27b) \ V0f) \ V1x) = (ap \ V0f \ V1x)))) \quad (45)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p \ V0t)) \Leftrightarrow (p \ V0t))) \quad (46)$$

Assume the following.

$$(\forall V0t \in 2.(((p \ V0t) \Rightarrow False) \Rightarrow (\neg(p \ V0t)))) \quad (47)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p \ V0t)) \Rightarrow ((p \ V0t) \Rightarrow False))) \quad (48)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (49)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (50)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (51)$$



Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (52)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (53)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (54)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (55)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap \ (ap \ (ap \ (c\_2Ebool\_2ECOND \ A\_27a) \ c\_2Ebool\_2ET) \ V0t1) \ V1t2) = V0t1) \wedge ((ap \ (ap \ (ap \ (c\_2Ebool\_2ECOND \ A\_27a) \ c\_2Ebool\_2EF) \ V0t1) \ V1t2) = V1t2)))) \quad (56)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p \ V0A) \vee (p \ V1B)) \Leftrightarrow ((p \ V1B) \vee (p \ V0A)))) \quad (57)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \wedge (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A)) \vee (\neg(p \ V1B)))) \wedge ((\neg((p \ V0A) \vee (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A)) \wedge (\neg(p \ V1B)))))) \quad (58)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p \ V0A) \Rightarrow (p \ V1B)) \Leftrightarrow ((\neg(p \ V0A)) \vee (p \ V1B)))) \quad (59)$$

Assume the following.

$$(\forall V0t \in 2.(((p \ V0t) \Rightarrow False) \Leftrightarrow ((p \ V0t) \Leftrightarrow False))) \quad (60)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \quad (61)$$

Assume the following.

$$2.(((p \ V0x) \Leftrightarrow (p \ V1x\_27)) \wedge ((p \ V1x\_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y\_27)))) \Rightarrow \\ (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x\_27) \Rightarrow (p \ V3y\_27)))) \quad (62)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ (\forall V2x \in A\_27a. (\forall V3x\_27 \in A\_27a. (\forall V4y \in A\_27a. \\ (\forall V5y\_27 \in A\_27a. (((p \ V0P) \Leftrightarrow (p \ V1Q)) \wedge ((p \ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\ ((\neg(p \ V1Q)) \Rightarrow (V4y = V5y\_27)))) \Rightarrow ((ap \ (ap \ (ap \ (c\_2Ebool\_2ECOND \ A\_27a) \\ V0P) \ V2x) \ V4y) = (ap \ (ap \ (ap \ (c\_2Ebool\_2ECOND \ A\_27a) \ V1Q) \ V3x\_27) \\ V5y\_27)))))))))) \quad (63)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow ((\forall V0t1 \in A\_27a. (\forall V1t2 \in \\ A\_27a. ((ap \ (ap \ (ap \ (c\_2Ebool\_2ECOND \ A\_27a) \ c\_2Ebool\_2ET) \ V0t1) \\ V1t2) = V0t1))) \wedge (\forall V2t1 \in A\_27a. (\forall V3t2 \in A\_27a. ((ap \\ (ap \ (ap \ (c\_2Ebool\_2ECOND \ A\_27a) \ c\_2Ebool\_2EF) \ V2t1) \ V3t2) = V3t2)))))) \quad (64)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((ap \ (c\_2Ecombin\_2EI \\ A\_27a) \ V0x) = V0x)) \quad (65)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (p \ (ap \ (ap \ c\_2Earithmetic\_2E\_3C\_3D \\ (ap \ c\_2Earithmetic\_2ENUMERAL \ (ap \ c\_2Earithmetic\_2EBIT1 \ c\_2Earithmetic\_2EZERO)))) \\ (ap \ (c\_2Efcp\_2Edimindex \ A\_27a) \ (c\_2Ebool\_2Ethe\_value \ A\_27a)))) \quad (66)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow \forall A\_27b.nonempty \ A\_27b \Rightarrow ( \\ \forall V0g \in (A\_27a^{ty\_2Enum\_2Enum}). (\forall V1i \in ty\_2Enum\_2Enum. \\ ((p \ (ap \ (ap \ c\_2Eprim\_rec\_2E\_3C \ V1i) \ (ap \ (c\_2Efcp\_2Edimindex \ A\_27b) \\ (c\_2Ebool\_2Ethe\_value \ A\_27b)))) \Rightarrow ((ap \ (ap \ (c\_2Efcp\_2Efcp\_index \\ A\_27a \ A\_27b) \ (ap \ (c\_2Efcp\_2EFCP \ A\_27a \ A\_27b) \ V0g)) \ V1i) = (ap \ V0g \\ V1i)))))) \quad (67)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\
& (ap\ (c.2EfcP.2Edimindex\ (ty\_2Esum\_2Esum\ A.27a\ A.27b))\ (c.2Ebool.2Ethe\_value \\
& (ty\_2Esum\_2Esum\ A.27a\ A.27b))) = (ap\ (ap\ (ap\ (c.2Ebool.2ECOND\ ty\_2Enum\_2Enum) \\
& (ap\ (ap\ c.2Ebool.2E\_2F\_5C\ (ap\ (c.2Epred\_set\_2EFINITE\ A.27a) \\
& (c.2Epred\_set\_2EUNIV\ A.27a)))\ (ap\ (c.2Epred\_set\_2EFINITE \\
& A.27b)\ (c.2Epred\_set\_2EUNIV\ A.27b))))\ (ap\ (ap\ c.2Earithmetic.2E\_2B \\
& (ap\ (c.2EfcP.2Edimindex\ A.27a)\ (c.2Ebool.2Ethe\_value\ A.27a))) \\
& (ap\ (c.2EfcP.2Edimindex\ A.27b)\ (c.2Ebool.2Ethe\_value\ A.27b)))) \\
& (ap\ c.2Earithmetic.2ENUMERAL\ (ap\ c.2Earithmetic.2EBIT1\ c.2Earithmetic.2EZERO)))) \\
& (68)
\end{aligned}$$

$$\begin{aligned} & ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmic\_2E\_2B \\ & c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap \\ & (ap c\_2Earithmic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in \\ & ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmic\_2E\_2B \\ & (ap c\_2Earithmic\_2ENUMERAL V2n)) (ap c\_2Earithmic\_2ENUMERAL \\ & V3m))) = (ap c\_2Earithmic\_2ENUMERAL (ap c\_2Enumeral\_2EiZ (ap \\ & (ap c\_2Earithmic\_2E\_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum. \\ & ((ap (ap c\_2Earithmic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge \\ & ((\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmic\_2E\_2A \\ & V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum. \\ & (\forall V7m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmic\_2E\_2A ( \\ & ap c\_2Earithmic\_2ENUMERAL V6n)) (ap c\_2Earithmic\_2ENUMERAL \\ & V7m))) = (ap c\_2Earithmic\_2ENUMERAL (ap (ap c\_2Earithmic\_2E\_2A \\ & V6n) V7m)))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmic\_2E\_2D \\ & c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum. \\ & ((ap (ap c\_2Earithmic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in \\ & ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmic\_2E\_2D \\ & (ap c\_2Earithmic\_2ENUMERAL V10n)) (ap c\_2Earithmic\_2ENUMERAL \\ & V11m))) = (ap c\_2Earithmic\_2ENUMERAL (ap (ap c\_2Earithmic\_2E\_2D \\ & V10n) V11m)))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmic\_2EEXP \\ & c\_2Enum\_2E0) (ap c\_2Earithmic\_2ENUMERAL (ap c\_2Earithmic\_2EBIT1 \\ & V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap \\ & (ap c\_2Earithmic\_2EEXP c\_2Enum\_2E0) (ap c\_2Earithmic\_2ENUMERAL \\ & (ap c\_2Earithmic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in \\ & ty\_2Enum\_2Enum.((ap (ap c\_2Earithmic\_2EEXP V14n) c\_2Enum\_2E0) = \\ & (ap c\_2Earithmic\_2ENUMERAL (ap c\_2Earithmic\_2EBIT1 c\_2Earithmic\_2EZERO)))))) \wedge \\ & ((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum. \\ & ((ap (ap c\_2Earithmic\_2EEXP (ap c\_2Earithmic\_2ENUMERAL V15n)) \\ & (ap c\_2Earithmic\_2ENUMERAL V16m))) = (ap c\_2Earithmic\_2ENUMERAL \\ & (ap (ap c\_2Earithmic\_2EEXP V15n) V16m)))))) \wedge (((ap c\_2Enum\_2ESUC \\ & c\_2Enum\_2E0) = (ap c\_2Earithmic\_2ENUMERAL (ap c\_2Earithmic\_2EBIT1 \\ & c\_2Earithmic\_2EZERO))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum.( \\ & (ap c\_2Enum\_2ESUC (ap c\_2Earithmic\_2ENUMERAL V17n)) = (ap c\_2Earithmic\_2ENUMERAL \\ & (ap c\_2Enum\_2ESUC V17n)))))) \wedge (((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = \\ & c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE \\ & (ap c\_2Earithmic\_2ENUMERAL V18n)) = (ap c\_2Earithmic\_2ENUMERAL \\ & (ap c\_2Eprim\_rec\_2EPRE V18n)))))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum. \\ & (((ap c\_2Earithmic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmic\_2EZERO)))))) \wedge \\ & ((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0 = (ap c\_2Earithmic\_2ENUMERAL \\ & V20n)) \Leftrightarrow (V20n = c\_2Earithmic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum. \\ & (\forall V22m \in ty\_2Enum\_2Enum.(((ap c\_2Earithmic\_2ENUMERAL \\ & V21n) = (ap c\_2Earithmic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))))) \wedge \\ & ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\ & V23n) c\_2Enum\_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty\_2Enum\_2Enum. \\ & ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmic\_2ENUMERAL \\ & V24n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmic\_2EZERO) \\ & V24n)))))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum. \\ & ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmic\_2ENUMERAL \\ & V25n)) (ap c\_2Earithmic\_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\ & V25n) V26m)))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmic\_2E\_3E \\ & c\_2Enum\_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty\_2Enum\_2Enum. \\ & ((p (ap (ap c\_2Earithmic\_2E\_3E (ap c\_2Earithmic\_2ENUMERAL \\ & V28n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmic\_2EZERO) \\ & V28n)))))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum. \\ & ((p (ap (ap c\_2Earithmic\_2E\_3E (ap c\_2Earithmic\_2ENUMERAL \\ & V29n)) (ap c\_2Earithmic\_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\ & V30m) V29n)))))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmic\_2E\_3C\_3D \\ & c\_2Enum\_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty\_2Enum\_2Enum. \\ & ((p (ap (ap c\_2Earithmic\_2E\_3C\_3D (ap c\_2Earithmic\_2ENUMERAL \\ & V32n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C\_3D c\_2Enum\_2E0) \\ & V32n)))))) \end{aligned}$$

[illegible]

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D c\_2Earithmetic\_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) c\_2Earithmetic\_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& (ap c\_2Earithmetic\_2EBIT2 V0n)) c\_2Earithmetic\_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow (\neg (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V1m) V0n)))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))))))))))
\end{aligned} \tag{71}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{72}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{73}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{74}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{75}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{76}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg( \\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{77}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{78}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p)))))))))) \\
& \tag{79}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge (( \\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p)))))))))) \\
& \tag{80}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p)))))) \\
& \tag{81}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a. nonempty \ A\_27a \Rightarrow \forall A\_27b. nonempty \ A\_27b \Rightarrow ( \\
& \forall V0w \in (ty\_2Efc\_2Ecart \ 2 \ A\_27a). (\forall V1i \in ty\_2Enum\_2Enum. \\
& ((p \ (ap \ (ap \ c\_2Eprim\_rec\_2E\_3C \ V1i) \ (ap \ (c\_2Efc\_2Edimindex \ A\_27b) \\
& (c\_2Ebool\_2Ethe\_value \ A\_27b)))) \Rightarrow ((p \ (ap \ (ap \ (c\_2Efc\_2Efc\_index \\
& 2 \ A\_27b) \ (ap \ (c\_2Ewords\_2Ew2w \ A\_27a \ A\_27b) \ V0w)) \ V1i)) \Leftrightarrow ((p \ (ap \\
& (ap \ c\_2Eprim\_rec\_2E\_3C \ V1i) \ (ap \ (c\_2Efc\_2Edimindex \ A\_27a) \ ( \\
& c\_2Ebool\_2Ethe\_value \ A\_27a)))) \wedge (p \ (ap \ (ap \ (c\_2Efc\_2Efc\_index \\
& 2 \ A\_27a) \ V0w) \ V1i)))))) \\
& \tag{82}
\end{aligned}$$

### Theorem 1

$$\begin{aligned}
& \forall A\_27a. nonempty \ A\_27a \Rightarrow \forall A\_27b. nonempty \ A\_27b \Rightarrow ( \\
& \forall V0i \in ty\_2Enum\_2Enum. (\forall V1a \in (ty\_2Efc\_2Ecart \ 2 \\
& A\_27a). (\forall V2b \in (ty\_2Efc\_2Ecart \ 2 \ A\_27b). (((p \ (ap \ (c\_2Epred\_set\_2EFINITE \\
& A\_27a) \ (c\_2Epred\_set\_2EUNIV \ A\_27a))) \wedge ((p \ (ap \ (c\_2Epred\_set\_2EFINITE \\
& A\_27b) \ (c\_2Epred\_set\_2EUNIV \ A\_27b))) \wedge (p \ (ap \ (ap \ c\_2Eprim\_rec\_2E\_3C \\
& V0i) \ (ap \ (c\_2Efc\_2Edimindex \ (ty\_2Esum\_2Esum \ A\_27a \ A\_27b)) \ (c\_2Ebool\_2Ethe\_value \\
& (ty\_2Esum\_2Esum \ A\_27a \ A\_27b)))))) \Rightarrow ((p \ (ap \ (ap \ (c\_2Efc\_2Efc\_index \\
& 2 \ (ty\_2Esum\_2Esum \ A\_27a \ A\_27b)) \ (ap \ (ap \ (c\_2Ewords\_2Eword\_join \\
& A\_27a \ A\_27b) \ V1a) \ V2b)) \ V0i)) \Leftrightarrow (p \ (ap \ (ap \ (ap \ (c\_2Ebool\_2ECOND \ 2) \\
& (ap \ (ap \ c\_2Eprim\_rec\_2E\_3C \ V0i) \ (ap \ (c\_2Efc\_2Edimindex \ A\_27b) \\
& (c\_2Ebool\_2Ethe\_value \ A\_27b)))) \ (ap \ (ap \ (c\_2Efc\_2Efc\_index \\
& 2 \ A\_27b) \ V2b) \ V0i)) \ (ap \ (ap \ (c\_2Efc\_2Efc\_index \ 2 \ A\_27a) \ V1a) \\
& (ap \ (ap \ c\_2Earithmetric\_2E\_2D \ V0i) \ (ap \ (c\_2Efc\_2Edimindex \ A\_27b) \\
& (c\_2Ebool\_2Ethe\_value \ A\_27b)))))))))) \\
& \tag{83}
\end{aligned}$$