thm_2Ewords_2Eword__rrx__0 (TM-bYjCBLMtRy8BLDLWaxaHc6xVsimPMTHH6)

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Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \tag{2}$$

Definition 6 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 7 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{3}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{4}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{5}$$

Definition 9 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 10 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 11 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 12 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 13 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 14 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 15 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{6}$$

Definition 16 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 17 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 18 *We define* c_2Eprim__rec_2EPRE *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap$ (c_2Ebool_2E

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{7}$$

Definition 19 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{8}$$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{9}$$

Definition 20 *We define* c_2Enumeral_2EiZ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $ty\_2Efcp\_2Efinite\_\_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efcp\_2Efinite\_\_image\ A0) \tag{10}$$

Let $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ebool\_2Eitself\ A0) \tag{11}$$

Let $c\_2Ebool\_2Ethe\_\_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2Ethe\_\_value\ A\_27a \in (ty\_2Ebool\_2Eitself\ A\_27a) \quad (12)$$

Let $c\_2Efcp\_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Efcp\_2Edimindex\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (13)$$

Definition 21 *We define* c_2Ebool_2E_3F_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap$ c_2Ebool_2E_2F_5C

Definition 22 *We define* c_2Efcp_2Efinite__index *to be* $\lambda A\_27a : \iota.(ap$ (c_2Emin_2E_40 $(A\_27a^{ty\_2Enum\_2Enu}$

Let $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efcp\_2Ecart\ A0\ A1) \quad (14)$$

Let $c\_2Efcp\_2Edest\_\_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efcp\_2Edest\_\_cart\ A\_27a\ A\_27b \in ((A\_27a^{(ty\_2Efcp\_2Efinite\_\_image\ A\_27b)})^{(ty\_2Efcp\_2Ecart\ A\_27a\ A\_27b)}) \quad (15)$$

Definition 23 *We define* c_2Efcp_2Efcp__index *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in (ty\_2Efcp\_2Ecart\ A\_27$

Definition 24 *We define* c_2Efcp_2EFCP *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(ap$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (16)$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \quad (17)$$

Definition 25 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Ewords\_2Eword\_\_rrx : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2Eword\_\_rrx\ A\_27a \in ((ty\_2Epair\_2Eprod\ 2\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a))^{(ty\_2Epair\_2Eprod\ 2\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a))}) \quad (18)$$

Let $c\_2Earithmetic\_2EODD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (19)$$

Definition 26 *We define* c_2Ewords_2Eword__lsb *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap$

Definition 27 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Definition 28 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 29 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{20}$$

Let $c\_2Ewords\_2EINT\_\_MIN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2EINT\_\_MIN\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27} \tag{21}$$

Let $c\_2Earithmetic\_2EDIV : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{22}$$

Definition 30 *We define* c_2Ebit_2EDIV__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Let $c\_2Earithmetic\_2EMOD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{23}$$

Definition 31 *We define* c_2Ebit_2EMOD__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2E$

Definition 32 *We define* c_2Ebit_2EBITS *to be* $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V$

Definition 33 *We define* c_2Ebit_2EBIT *to be* $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap$

Definition 34 *We define* c_2Ewords_2En2w *to be* $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.(ap$ (c_2Efcp_2EFC

Definition 35 *We define* c_2Ewords_2Eword__L *to be* $\lambda A\_27a : \iota.(ap$ (c_2Ewords_2En2w $A\_27a)\ (ap\ (c\_2Ewo$

Definition 36 *We define* c_2Ewords_2Eword__ror *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).\lambda V1$

Definition 37 *We define* c_2Ewords_2Eword__rol *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).\lambda V1$

Definition 38 *We define* c_2Ewords_2Eword__lsr *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).\lambda V1$

Definition 39 *We define* c_2Ewords_2Eword__msb *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap$

Definition 40 *We define* c_2Ewords_2Eword__asr *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).\lambda V1$

Definition 41 *We define* c_2Ewords_2Eword__lsl *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).\lambda V1$

Assume the following.

$$True \tag{24}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{25}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{26}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg (p\ V0t)))) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{28}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{29}$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{30}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{31}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{32}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{33}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{34}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{35}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V0t1)\ V1t2) = V1t2)))) \tag{36}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{37}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{38}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in (ty\_2Efcp\_2Ecart\ A\_27a\ A\_27b).(\forall V1y \in (ty\_2Efcp\_2Ecart\ A\_27a\ A\_27b).((V0x = V1y) \Leftrightarrow (\forall V2i \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V2i)\ (ap\ (c\_2Efcp\_2Edimindex\ A\_27b)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27b)))) \Rightarrow ((ap\ (ap\ (\mathsf{c\_2Efcp\_2Efcp\_\_index}\ A\_27a\ A\_27b)\ V0x)\ V2i) = (ap\ (ap\ (\mathsf{c\_2Efcp\_2Efcp\_\_index}\ A\_27a\ A\_27b)\ V1y)\ V2i))))))) \tag{39}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(\forall V1i \in ty\_2Enum\_2Enum. ((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1i)\ (ap\ (c\_2Efcp\_2Edimindex\ A\_27b)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27b)))) \Rightarrow ((ap\ (ap\ (\mathsf{c\_2Efcp\_2Efcp\_\_index}\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Efcp\_2EFCP}\ A\_27a\ A\_27b)\ V0g))\ V1i) = (ap\ V0g\ V1i))))) \tag{40}$$

Assume the following.

$$((\forall V0n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Enum\_2E0})\ V0n) = V0n))\wedge((\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ \mathsf{c\_2Enum\_2E0}) = V1n))\wedge((\forall V2n \in ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V2n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V3m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V2n)\ V3m))))))\wedge((\forall V4n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \mathsf{c\_2Enum\_2E0})\ V4n) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V5n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V5n)\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V6n \in ty\_2Enum\_2Enum.(\forall V7m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V6n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V7m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V6n)\ V7m)))))\wedge((\forall V8n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ \mathsf{c\_2Enum\_2E0})\ V8n) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V9n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V9n)\ \mathsf{c\_2Enum\_2E0}) = V9n))\wedge((\forall V10n \in ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V10n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V11m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V10n)\ V11m)))))\wedge((\forall V12n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ \mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V12n))) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V13n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ \mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V13n))) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V14n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ V14n)\ \mathsf{c\_2Enum\_2E0}) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\wedge((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V15n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V16m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2EEXP\ V15n)\ V16m)))))\wedge(((ap\ \mathsf{c\_2Enum\_2ESUC}\ \mathsf{c\_2Enum\_2E0}) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V17n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V17n)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V17n))))\wedge(((ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0})\wedge((\forall V18n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V18n)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ V18n))))\wedge((\forall V19n \in ty\_2Enum\_2Enum.(((ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V19n) = \mathsf{c\_2Enum\_2E0})\Leftrightarrow(V19n = \mathsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V20n \in ty\_2Enum\_2Enum.((\mathsf{c\_2Enum\_2E0} = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V20n))\Leftrightarrow(V20n = \mathsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V21n \in ty\_2Enum\_2Enum.(\forall V22m \in ty\_2Enum\_2Enum.(((ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V21n) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V22m))\Leftrightarrow(V21n = V22m))))\wedge((\forall V23n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V23n)\ \mathsf{c\_2Enum\_2E0}))\Leftrightarrow False))\wedge((\forall V24n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V24n)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})\ V24n))))\wedge((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V25n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V26m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V25n)\ V26m)))))\wedge((\forall V27n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E}\ \mathsf{c\_2Enum\_2E0})\ V27n))\Leftrightarrow False))\wedge((\forall V28n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V28n))\ \mathsf{c\_2Enum\_2E0}))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})\ V28n))))\wedge((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V29n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V30m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V30m)\ V29n)))))\wedge((\forall V31n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ \mathsf{c\_2Enum\_2E0})\ V31n))\Leftrightarrow True))\wedge((\forall V32n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.(\forall V2a \in A\_27a.(\forall V3b \in A\_27b.(((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y) = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))) \tag{42}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0c \in 2.(\forall V1w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).((ap\ (c\_2Ewords\_2Eword\_\_rrx\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ 2\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a))\ V0c)\ V1w)) = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ 2\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a))\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_lsb}\ A\_27a)\ V1w))\ (ap\ (\mathsf{c\_2Efcp\_2EFCP}\ 2\ A\_27a)\ (\lambda V2i \in ty\_2Enum\_2Enum. (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ 2)\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ ty\_2Enum\_2Enum)\ V2i)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ (c\_2Efcp\_2Edimindex\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a)))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))\ V0c)\ (ap\ (ap\ (\mathsf{c\_2Efcp\_2Efcp\_\_index}\ 2\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_lsr}\ A\_27a)\ V1w)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ V2i)))))))) \tag{43}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum.((p\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_lsb}\ A\_27a)\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ V0n))) \Leftrightarrow (p\ (ap\ c\_2Earithmetic\_2EODD\ V0n)))) \tag{44}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0i \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0i)\ (ap\ (c\_2Efcp\_2Edimindex\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a)))) \Rightarrow (\neg (p\ (ap\ (ap\ (\mathsf{c\_2Efcp\_2Efcp\_\_index}\ 2\ A\_27a)\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ \mathsf{c\_2Enum\_2E0}))\ V0i))))) \tag{45}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ (ap\ (c\_2Efcp\_2Edimindex\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a)))) \Rightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Efcp\_2Efcp\_\_index}\ 2\ A\_27a)\ (\mathsf{c\_2Ewords\_2Eword\_\_L}\ A\_27a))\ V0n)) \Leftrightarrow (V0n = (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ (c\_2Efcp\_2Edimindex\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a)))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))))) \tag{46}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0n \in ty\_2Enum\_2Enum.((ap\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_lsl}\ A\_27a)\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ \mathsf{c\_2Enum\_2E0}))\ V0n) = (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ \mathsf{c\_2Enum\_2E0}))) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_asr}\ A\_27a)\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ \mathsf{c\_2Enum\_2E0}))\ V1n) = (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ \mathsf{c\_2Enum\_2E0}))) \wedge ((\forall V2n \in ty\_2Enum\_2Enum.((ap\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_lsr}\ A\_27a)\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ \mathsf{c\_2Enum\_2E0}))\ V2n) = (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ \mathsf{c\_2Enum\_2E0}))) \wedge ((\forall V3n \in ty\_2Enum\_2Enum.((ap\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_rol}\ A\_27a)\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ \mathsf{c\_2Enum\_2E0}))\ V3n) = (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ \mathsf{c\_2Enum\_2E0}))) \wedge (\forall V4n \in ty\_2Enum\_2Enum.((ap\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_ror}\ A\_27a)\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ \mathsf{c\_2Enum\_2E0}))\ V4n) = (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ \mathsf{c\_2Enum\_2E0})))))))) \tag{47}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (((ap\ (c\_2Ewords\_2Eword\_\_rrx\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ 2\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a))\ \mathsf{c\_2Ebool\_2EF})\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ \mathsf{c\_2Enum\_2E0}))) = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ 2\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a))\ \mathsf{c\_2Ebool\_2EF})\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ \mathsf{c\_2Enum\_2E0}))) \wedge ((ap\ (c\_2Ewords\_2Eword\_\_rrx\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ 2\ (ty\_2Efcp\_2Ecart\ 2\ A\_27b))\ \mathsf{c\_2Ebool\_2ET})\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27b)\ \mathsf{c\_2Enum\_2E0}))) = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ 2\ (ty\_2Efcp\_2Ecart\ 2\ A\_27b))\ \mathsf{c\_2Ebool\_2EF})\ (\mathsf{c\_2Ewords\_2Eword\_\_L}\ A\_27b))))$$