

l29\_nat\_lat  
(TMKJ4TfxPTiBcGcp8tTixg5di38G4poauC7)

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Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $u1\_struct\_0 : \iota \Rightarrow \iota$  be given. Let  $k10\_nat\_lat : \iota$  be given. Let  $k2\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v7\_ordinal1 : \iota \Rightarrow o$  be given. Let  $k6\_nat\_d : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k5\_nat\_d : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v2\_struct\_0 : \iota \Rightarrow o$  be given. Let  $v3\_lattices : \iota \Rightarrow o$  be given. Let  $l3\_lattices : \iota \Rightarrow o$  be given. Let  $l1\_lattices : \iota \Rightarrow o$  be given. Let  $l2\_lattices : \iota \Rightarrow o$  be given. Let  $v1\_xboole\_0 : \iota \Rightarrow o$  be given. Assume the following.

$$\forall X0.(v7\_ordinal1 X0) \Rightarrow (\forall X1.(v7\_ordinal1 X1) \Rightarrow (k6\_nat\_d X0 (k5\_nat\_d X0 X1) = X0)) \quad (1)$$

Assume the following.

$$\forall X0.(v7\_ordinal1 X0) \Rightarrow (\forall X1.(v7\_ordinal1 X1) \Rightarrow (\forall X2.(v7\_ordinal1 X2) \Rightarrow (k6\_nat\_d X0 (k6\_nat\_d X1 X2) = k6\_nat\_d (k6\_nat\_d X0 X1) X2))) \quad (2)$$

Assume the following.

$$\forall X0.\forall X1.((m1\_subset\_1 X0 (u1\_struct\_0 k10\_nat\_lat)) \wedge (m1\_subset\_1 X1 (u1\_struct\_0 k10\_nat\_lat))) \Rightarrow (k2\_lattices k10\_nat\_lat X0 X1 = k6\_nat\_d X0 X1) \quad (3)$$

Assume the following.

$$\forall X0.\forall X1.((m1\_subset\_1 X0 (u1\_struct\_0 k10\_nat\_lat)) \wedge (m1\_subset\_1 X1 (u1\_struct\_0 k10\_nat\_lat))) \Rightarrow (k1\_lattices k10\_nat\_lat X0 X1 = k5\_nat\_d X0 X1) \quad (4)$$

Assume the following.

$$(\neg v2\_struct\_0 k10\_nat\_lat) \wedge (v3\_lattices k10\_nat\_lat) \quad (5)$$

Assume the following.

$$\forall X0.(l3\_lattices X0) \Rightarrow ((l1\_lattices X0) \wedge (l2\_lattices X0)) \quad (6)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge(l1\_lattices X0))\wedge((m1\_subset\_1 X1 (u1\_struct\_0 X0))\wedge(m1\_subset\_1 X2 (u1\_struct\_0 X0))))\Rightarrow(m1\_subset\_1 (k2\_lattices X0 X1 X2) (u1\_struct\_0 X0)) \quad (7)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge(l2\_lattices X0))\wedge((m1\_subset\_1 X1 (u1\_struct\_0 X0))\wedge(m1\_subset\_1 X2 (u1\_struct\_0 X0))))\Rightarrow(m1\_subset\_1 (k1\_lattices X0 X1 X2) (u1\_struct\_0 X0)) \quad (8)$$

Assume the following.

$$(v3\_lattices k10\_nat\_lat)\wedge(l3\_lattices k10\_nat\_lat) \quad (9)$$

Assume the following.

$$\forall X0.(m1\_subset\_1 X0 (u1\_struct\_0 k10\_nat\_lat))\Rightarrow((\neg v1\_xboole\_0 X0)\wedge(v7\_ordinal1 X0)) \quad (10)$$

**Theorem 1**

$$\begin{aligned} & (\forall X0.(m1\_subset\_1 X0 (u1\_struct\_0 k10\_nat\_lat))\Rightarrow(\forall X1. \\ & (m1\_subset\_1 X1 (u1\_struct\_0 k10\_nat\_lat))\Rightarrow(\forall X2.(m1\_subset\_1 \\ & X2 (u1\_struct\_0 k10\_nat\_lat))\Rightarrow(k2\_lattices k10\_nat\_lat X0 (k2\_lattices \\ & k10\_nat\_lat X1 X2) = k2\_lattices k10\_nat\_lat (k2\_lattices k10\_nat\_lat \\ & X0 X1) X2))))\wedge(\forall X0.(m1\_subset\_1 X0 (u1\_struct\_0 k10\_nat\_lat))\Rightarrow \\ & (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 k10\_nat\_lat))\Rightarrow(k2\_lattices \\ & k10\_nat\_lat X0 (k1\_lattices k10\_nat\_lat X0 X1) = X0))) \end{aligned}$$