

# l42\_substlat

(TMHEPpSsQq46esJ2p5zTzg8ydzEmftXkzQp)

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Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $u1\_struct\_0 : \iota \Rightarrow \iota$  be given. Let  $k5\_substlat : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k2\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k5\_finsub\_1 : \iota \Rightarrow \iota$  be given. Let  $k4\_partfun1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k3\_substlat : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k4\_substlat : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $m2\_subset\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k1\_substlat : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v1\_xboole\_0 : \iota \Rightarrow o$  be given. Let  $k1\_zfmisc\_1 : \iota \Rightarrow \iota$  be given. Let  $v1\_funct\_1 : \iota \Rightarrow o$  be given. Let  $v1\_funct\_2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k2\_zfmisc\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k5\_binop\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_binop\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $u1\_lattices : \iota \Rightarrow \iota$  be given. Let  $u2\_lattices : \iota \Rightarrow \iota$  be given. Let  $k2\_substlat : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v2\_struct\_0 : \iota \Rightarrow o$  be given. Let  $v3\_lattices : \iota \Rightarrow o$  be given. Let  $v4\_finsub\_1 : \iota \Rightarrow o$  be given. Let  $l2\_lattices : \iota \Rightarrow o$  be given. Let  $l1\_lattices : \iota \Rightarrow o$  be given. Let  $l3\_lattices : \iota \Rightarrow o$  be given. Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (m1\_subset\_1 X2 (k5\_finsub\_1 \\ & (k4\_partfun1 X0 X1))) \Rightarrow (k3\_substlat X0 X1 (k4\_substlat X0 X1 X2 X2) = \\ & \quad k3\_substlat X0 X1 X2) \end{aligned} \tag{1}$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (m2\_subset\_1 X2 (k5\_finsub\_1 \\ & (k4\_partfun1 X0 X1)) (k1\_substlat X0 X1)) \Rightarrow (k3\_substlat X0 X1 X2 = \\ & \quad X2) \end{aligned} \tag{2}$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((\neg v1\_xboole\_0 X0) \wedge ((\neg v1\_xboole\_0 X1) \wedge \\ & (m1\_subset\_1 X1 (k1\_zfmisc\_1 X0)))) \Rightarrow (\forall X2. (m2\_subset\_1 \\ & \quad X2 X0 X1) \Leftrightarrow (m1\_subset\_1 X2 X1)) \end{aligned} \tag{3}$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.\forall X3.(((v1\_funct\_1 X1)\wedge \\ & ((v1\_funct\_2 X1 (k2\_zfmisc\_1 X0 X0) X0)\wedge(m1\_subset\_1 X1 (k1\_zfmisc\_1 \\ & (k2\_zfmisc\_1 (k2\_zfmisc\_1 X0 X0) X0))))\wedge((m1\_subset\_1 X2 X0)\wedge \\ & (m1\_subset\_1 X3 X0)))\Rightarrow(k5\_binop\_1 X0 X1 X2 X3 = k1\_binop\_1 X1 X2 X3) \end{aligned} \quad (4)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(m2\_subset\_1 X2 (k5\_finsub\_1 \\ & (k4\_partfun1 X0 X1)) (k1\_substlat X0 X1))\Rightarrow(\forall X3.(m2\_subset\_1 \\ & X3 (k5\_finsub\_1 (k4\_partfun1 X0 X1)) (k1\_substlat X0 X1))\Rightarrow(\forall X4. \\ & (m2\_subset\_1 X4 (k5\_finsub\_1 (k4\_partfun1 X0 X1)) (k1\_substlat \\ & X0 X1))\Rightarrow(k1\_binop\_1 (u1\_lattices (k5\_substlat X0 X1)) X2 (k1\_binop\_1 \\ & (u2\_lattices (k5\_substlat X0 X1)) X3 X4) = k1\_binop\_1 (u2\_lattices \\ & (k5\_substlat X0 X1)) (k1\_binop\_1 (u1\_lattices (k5\_substlat X0 \\ & X1)) X2 X3) (k1\_binop\_1 (u1\_lattices (k5\_substlat X0 X1)) X2 X4)))) \end{aligned} \quad (5)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 \\ & (k5\_substlat X0 X1)))\Rightarrow(\forall X3.(m1\_subset\_1 X3 (u1\_struct\_0 \\ & (k5\_substlat X0 X1)))\Rightarrow(k2\_lattices (k5\_substlat X0 X1) X2 X3 = k2\_lattices \\ & (k5\_substlat X0 X1) X3 X2)) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 \\ & (k5\_substlat X0 X1)))\Rightarrow(\forall X3.(m1\_subset\_1 X3 (u1\_struct\_0 \\ & (k5\_substlat X0 X1)))\Rightarrow(k1\_lattices (k5\_substlat X0 X1) (k2\_lattices \\ & (k5\_substlat X0 X1) X2 X3) X3 = X3)) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 \\ & (k5\_substlat X0 X1)))\Rightarrow(\forall X3.(m1\_subset\_1 X3 (u1\_struct\_0 \\ & (k5\_substlat X0 X1)))\Rightarrow(k1\_lattices (k5\_substlat X0 X1) X2 X3 = k1\_lattices \\ & (k5\_substlat X0 X1) X3 X2)) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.\forall X3.((m1\_subset\_1 X2 \\ & (k1\_substlat X0 X1))\wedge(m1\_subset\_1 X3 (k1\_substlat X0 X1)))\Rightarrow(k2\_substlat \\ & X0 X1 X2 X2 = X2) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.(\neg v2\_struct\_0 (k5\_substlat X0 X1))\wedge(v3\_lattices \\ & (k5\_substlat X0 X1)) \end{aligned} \quad (10)$$

Assume the following.

$$\forall X0.(\neg v1\_xboole\_0 (k5\_finsub\_1 X0)) \wedge (v4\_finsub\_1 (k5\_finsub\_1 X0)) \quad (11)$$

Assume the following.

$$\forall X0.\forall X1.\neg v1\_xboole\_0 (k1\_substlat X0 X1) \quad (12)$$

Assume the following.

$$\begin{aligned} \forall X0.(l2\_lattices X0) \Rightarrow & ((v1\_funct\_1 (u2\_lattices X0)) \wedge \\ & ((v1\_funct\_2 (u2\_lattices X0) (k2\_zfmisc\_1 (u1\_struct\_0 X0) ( \\ & u1\_struct\_0 X0)) (u1\_struct\_0 X0)) \wedge (m1\_subset\_1 (u2\_lattices \\ & X0) (k1\_zfmisc\_1 (k2\_zfmisc\_1 (k2\_zfmisc\_1 (u1\_struct\_0 X0) ( \\ & u1\_struct\_0 X0)) (u1\_struct\_0 X0)))))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} \forall X0.(l1\_lattices X0) \Rightarrow & ((v1\_funct\_1 (u1\_lattices X0)) \wedge \\ & ((v1\_funct\_2 (u1\_lattices X0) (k2\_zfmisc\_1 (u1\_struct\_0 X0) ( \\ & u1\_struct\_0 X0)) (u1\_struct\_0 X0)) \wedge (m1\_subset\_1 (u1\_lattices \\ & X0) (k1\_zfmisc\_1 (k2\_zfmisc\_1 (k2\_zfmisc\_1 (u1\_struct\_0 X0) ( \\ & u1\_struct\_0 X0)) (u1\_struct\_0 X0)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\forall X0.(l3\_lattices X0) \Rightarrow ((l1\_lattices X0) \wedge (l2\_lattices X0)) \quad (15)$$

Assume the following.

$$\forall X0.\forall X1.(v3\_lattices (k5\_substlat X0 X1)) \wedge (l3\_lattices (k5\_substlat X0 X1)) \quad (16)$$

Assume the following.

$$\begin{aligned} \forall X0.\forall X1.\forall X2.\forall X3. & ((m1\_subset\_1 X2 \\ & (k1\_substlat X0 X1)) \wedge (m1\_subset\_1 X3 (k1\_substlat X0 X1))) \Rightarrow (m1\_subset\_1 \\ & (k2\_substlat X0 X1 X2 X3) (k5\_finsub\_1 (k4\_partfun1 X0 X1))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} \forall X0.\forall X1.\forall X2. & (((\neg v2\_struct\_0 X0) \wedge (l1\_lattices \\ & X0)) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge (m1\_subset\_1 X2 (u1\_struct\_0 \\ & X0)))) \Rightarrow (m1\_subset\_1 (k2\_lattices X0 X1 X2) (u1\_struct\_0 X0)) \end{aligned} \quad (18)$$

Assume the following.

$$\forall X0.\forall X1.m1\_subset\_1 (k1\_substlat X0 X1) (k1\_zfmisc\_1 (k5\_finsub\_1 (k4\_partfun1 X0 X1))) \quad (19)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge(l2\_lattices \\ & X0))\wedge((m1\_subset\_1 X1 (u1\_struct\_0 X0))\wedge(m1\_subset\_1 X2 (u1\_struct\_0 \\ & X0))))\Rightarrow(m1\_subset\_1 (k1\_lattices X0 X1 X2) (u1\_struct\_0 X0)) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.((v3\_lattices X2)\wedge(l3\_lattices \\ & X2))\Rightarrow((X2 = k5\_substlat X0 X1)\Leftrightarrow((u1\_struct\_0 X2 = k1\_substlat X0 \\ & X1)\wedge(\forall X3.(m2\_subset\_1 X3 (k5\_finsub\_1 (k4\_partfun1 X0 \\ & X1)) (k1\_substlat X0 X1))\Rightarrow(\forall X4.(m2\_subset\_1 X4 (k5\_finsub\_1 \\ & (k4\_partfun1 X0 X1)) (k1\_substlat X0 X1))\Rightarrow((k1\_binop\_1 (u2\_lattices \\ & X2) X3 X4 = k3\_substlat X0 X1 (k2\_substlat X0 X1 X3 X4))\wedge(k1\_binop\_1 \\ & (u1\_lattices X2) X3 X4 = k3\_substlat X0 X1 (k4\_substlat X0 X1 X3 X4))))))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & \forall X0.((\neg v2\_struct\_0 X0)\wedge(l1\_lattices X0))\Rightarrow(\forall X1. \\ & (m1\_subset\_1 X1 (u1\_struct\_0 X0))\Rightarrow(\forall X2.(m1\_subset\_1 X2 \\ & (u1\_struct\_0 X0))\Rightarrow(k2\_lattices X0 X1 X2 = k5\_binop\_1 (u1\_struct\_0 \\ & X0) (u1\_lattices X0) X1 X2))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & \forall X0.((\neg v2\_struct\_0 X0)\wedge(l2\_lattices X0))\Rightarrow(\forall X1. \\ & (m1\_subset\_1 X1 (u1\_struct\_0 X0))\Rightarrow(\forall X2.(m1\_subset\_1 X2 \\ & (u1\_struct\_0 X0))\Rightarrow(k1\_lattices X0 X1 X2 = k5\_binop\_1 (u1\_struct\_0 \\ & X0) (u2\_lattices X0) X1 X2))) \end{aligned} \quad (23)$$

**Theorem 1**

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 \\ & (k5\_substlat X0 X1)))\Rightarrow(\forall X3.(m1\_subset\_1 X3 (u1\_struct\_0 \\ & (k5\_substlat X0 X1)))\Rightarrow(k2\_lattices (k5\_substlat X0 X1) X2 (k1\_lattices \\ & (k5\_substlat X0 X1) X2 X3) = X2)) \end{aligned}$$