

l74\_polyred  
(TMRD9YmcqQKQN4PCZV9in2yFMMUeRqYA42J)

October 27, 2020

Let  $v3\_ordinal1 : \iota \Rightarrow o$  be given. Let  $v1\_partfun1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k15\_pre\_poly : \iota \Rightarrow \iota$  be given. Let  $v1\_relat\_2 : \iota \Rightarrow o$  be given. Let  $v4\_relat\_2 : \iota \Rightarrow o$  be given. Let  $v6\_relat\_2 : \iota \Rightarrow o$  be given. Let  $v8\_relat\_2 : \iota \Rightarrow o$  be given. Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k1\_zfmisc\_1 : \iota \Rightarrow \iota$  be given. Let  $k2\_zfmisc\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v7\_struct\_0 : \iota \Rightarrow o$  be given. Let  $v13\_algstr\_0 : \iota \Rightarrow o$  be given. Let  $v33\_algstr\_0 : \iota \Rightarrow o$  be given. Let  $v3\_group\_1 : \iota \Rightarrow o$  be given. Let  $v5\_group\_1 : \iota \Rightarrow o$  be given. Let  $v4\_vectsp\_1 : \iota \Rightarrow o$  be given. Let  $v5\_vectsp\_1 : \iota \Rightarrow o$  be given. Let  $v3\_rlvect\_1 : \iota \Rightarrow o$  be given. Let  $v4\_rlvect\_1 : \iota \Rightarrow o$  be given. Let  $l6\_algstr\_0 : \iota \Rightarrow o$  be given. Let  $v1\_funct\_1 : \iota \Rightarrow o$  be given. Let  $v1\_funct\_2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $u1\_struct\_0 : \iota \Rightarrow \iota$  be given. Let  $v1\_polynom1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $r4\_polyred : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k7\_polynom1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v1\_relat\_1 : \iota \Rightarrow o$  be given. Let  $v4\_relat\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v4\_valued\_0 : \iota \Rightarrow o$  be given. Let  $v2\_pre\_poly : \iota \Rightarrow o$  be given. Let  $r3\_polyred : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k2\_polynom1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k11\_pre\_poly : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k3\_termord : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $r2\_funct\_2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k6\_polynom1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k5\_polynom7 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_polyred : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k3\_vectsp\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k3\_polynom1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k4\_termord :$

$\iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned}
& \forall X0.(v3\_ordinal1\ X0) \Rightarrow (\forall X1.((v1\_partfun1\ X1\ (k15\_pre\_poly \\
& \quad X0)) \wedge ((v1\_relat\_2\ X1) \wedge ((v4\_relat\_2\ X1) \wedge ((v6\_relat\_2\ X1) \wedge (( \\
& v8\_relat\_2\ X1) \wedge (m1\_subset\_1\ X1\ (k1\_zfmisc\_1\ (k2\_zfmisc\_1\ (k15\_pre\_poly \\
& \quad X0)\ (k15\_pre\_poly\ X0)))))))))) \Rightarrow (\forall X2.((\neg v7\_struct\_0\ X2) \wedge \\
& ((v13\_algstr\_0\ X2) \wedge ((v33\_algstr\_0\ X2) \wedge ((v3\_group\_1\ X2) \wedge ((v5\_group\_1 \\
& \quad X2) \wedge ((v4\_vectsp\_1\ X2) \wedge ((v5\_vectsp\_1\ X2) \wedge ((v3\_rlvect\_1\ X2) \wedge \\
& \quad ((v4\_rlvect\_1\ X2) \wedge (l6\_algstr\_0\ X2)))))))))) \Rightarrow (\forall X3.(( \\
& \quad v1\_funct\_1\ X3) \wedge ((v1\_funct\_2\ X3\ (k15\_pre\_poly\ X0)\ (u1\_struct\_0 \\
& \quad X2)) \wedge ((v1\_polynom1\ X3\ (k15\_pre\_poly\ X0)\ X2) \wedge (m1\_subset\_1\ X3\ ( \\
& \quad k1\_zfmisc\_1\ (k2\_zfmisc\_1\ (k15\_pre\_poly\ X0)\ (u1\_struct\_0\ X2)))))) \Rightarrow \\
& \quad (\forall X4.((v1\_funct\_1\ X4) \wedge ((v1\_funct\_2\ X4\ (k15\_pre\_poly\ X0) \\
& (u1\_struct\_0\ X2)) \wedge ((v1\_polynom1\ X4\ (k15\_pre\_poly\ X0)\ X2) \wedge (m1\_subset\_1 \\
& \quad X4\ (k1\_zfmisc\_1\ (k2\_zfmisc\_1\ (k15\_pre\_poly\ X0)\ (u1\_struct\_0\ X2)))))) \Rightarrow \\
& \quad (\forall X5.((v1\_funct\_1\ X5) \wedge ((v1\_funct\_2\ X5\ (k15\_pre\_poly\ X0) \\
& (u1\_struct\_0\ X2)) \wedge ((v1\_polynom1\ X5\ (k15\_pre\_poly\ X0)\ X2) \wedge (m1\_subset\_1 \\
& \quad X5\ (k1\_zfmisc\_1\ (k2\_zfmisc\_1\ (k15\_pre\_poly\ X0)\ (u1\_struct\_0\ X2)))))) \Rightarrow \\
& \quad ((r4\_polyred\ X0\ X1\ X2\ X3\ X4\ X5) \Leftrightarrow (\exists X6.((v1\_relat\_1\ X6) \wedge (( \\
& v4\_relat\_1\ X6\ X0) \wedge ((v1\_funct\_1\ X6) \wedge ((v1\_partfun1\ X6\ X0) \wedge ((v4\_valued\_0 \\
& \quad X6) \wedge (v2\_pre\_poly\ X6)))))) \wedge (r3\_polyred\ X0\ X1\ X2\ X3\ X4\ X5\ X6)))))) \\
& \hspace{15em} (1)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall X0.(v3\_ordinal1\ X0) \Rightarrow (\forall X1.((v1\_partfun1\ X1\ (k15\_pre\_poly \\
& \quad X0)) \wedge ((v1\_relat\_2\ X1) \wedge ((v4\_relat\_2\ X1) \wedge ((v6\_relat\_2\ X1) \wedge (( \\
& \quad v8\_relat\_2\ X1) \wedge (m1\_subset\_1\ X1\ (k1\_zfmisc\_1\ (k2\_zfmisc\_1\ (k15\_pre\_poly \\
& \quad \quad X0)\ (k15\_pre\_poly\ X0)))))))))) \Rightarrow (\forall X2.((\neg v7\_struct\_0\ X2) \wedge \\
& \quad ((v13\_algstr\_0\ X2) \wedge ((v33\_algstr\_0\ X2) \wedge ((v3\_group\_1\ X2) \wedge ((v5\_group\_1 \\
& \quad \quad X2) \wedge ((v4\_vectsp\_1\ X2) \wedge ((v5\_vectsp\_1\ X2) \wedge ((v3\_rlvect\_1\ X2) \wedge \\
& \quad \quad \quad ((v4\_rlvect\_1\ X2) \wedge (l6\_algstr\_0\ X2)))))))))) \Rightarrow (\forall X3.(( \\
& \quad v1\_funct\_1\ X3) \wedge ((v1\_funct\_2\ X3\ (k15\_pre\_poly\ X0)\ (u1\_struct\_0 \\
& \quad \quad X2)) \wedge ((v1\_polynom1\ X3\ (k15\_pre\_poly\ X0)\ X2) \wedge (m1\_subset\_1\ X3\ ( \\
& \quad \quad k1\_zfmisc\_1\ (k2\_zfmisc\_1\ (k15\_pre\_poly\ X0)\ (u1\_struct\_0\ X2)))))) \Rightarrow \\
& \quad (\forall X4.((v1\_funct\_1\ X4) \wedge ((v1\_funct\_2\ X4\ (k15\_pre\_poly\ X0) \\
& \quad \quad (u1\_struct\_0\ X2)) \wedge ((v1\_polynom1\ X4\ (k15\_pre\_poly\ X0)\ X2) \wedge (m1\_subset\_1 \\
& \quad \quad X4\ (k1\_zfmisc\_1\ (k2\_zfmisc\_1\ (k15\_pre\_poly\ X0)\ (u1\_struct\_0\ X2)))))) \Rightarrow \\
& \quad (\forall X5.((v1\_funct\_1\ X5) \wedge ((v1\_funct\_2\ X5\ (k15\_pre\_poly\ X0) \\
& \quad \quad (u1\_struct\_0\ X2)) \wedge ((v1\_polynom1\ X5\ (k15\_pre\_poly\ X0)\ X2) \wedge (m1\_subset\_1 \\
& \quad \quad X5\ (k1\_zfmisc\_1\ (k2\_zfmisc\_1\ (k15\_pre\_poly\ X0)\ (u1\_struct\_0\ X2)))))) \Rightarrow \\
& \quad (\forall X6.((v1\_relat\_1\ X6) \wedge ((v4\_relat\_1\ X6\ X0) \wedge ((v1\_funct\_1 \\
& \quad \quad X6) \wedge ((v1\_partfun1\ X6\ X0) \wedge ((v4\_valued\_0\ X6) \wedge (v2\_pre\_poly\ X6)))))) \Rightarrow \\
& \quad ((r3\_polyred\ X0\ X1\ X2\ X3\ X4\ X5\ X6) \Leftrightarrow ((X3 \neq k7\_polynom1\ X0\ X2) \wedge ((X4 \neq \\
& \quad k7\_polynom1\ X0\ X2) \wedge ((X6 \in k2\_polynom1\ (k15\_pre\_poly\ X0)\ X2\ X3) \wedge \\
& \quad \quad (\exists X7.((v1\_relat\_1\ X7) \wedge ((v4\_relat\_1\ X7\ X0) \wedge ((v1\_funct\_1 \\
& \quad \quad X7) \wedge ((v1\_partfun1\ X7\ X0) \wedge ((v4\_valued\_0\ X7) \wedge (v2\_pre\_poly\ X7)))))) \wedge \\
& \quad \quad ((k11\_pre\_poly\ X0\ X7\ (k3\_termord\ X0\ X1\ X2\ X4) = X6) \wedge (r2\_funct\_2\ ( \\
& \quad \quad k15\_pre\_poly\ X0)\ (u1\_struct\_0\ X2)\ X5\ (k6\_polynom1\ X0\ X2\ X3\ (k5\_polynom7 \\
& \quad \quad X0\ X2\ (k1\_polyred\ X0\ X7\ X2\ X4)\ (k3\_vectsp\_1\ X2\ (k3\_polynom1\ X0\ X2\ X3 \\
& \quad \quad \quad X6)\ (k4\_termord\ X0\ X1\ X2\ X4))))))))))
\end{aligned} \tag{2}$$

**Theorem 1**

$$\begin{aligned}
& \forall X0.(v3\_ordinal1\ X0) \Rightarrow (\forall X1.((v1\_partfun1\ X1\ (k15\_pre\_poly \\
& \quad X0)) \wedge ((v1\_relat\_2\ X1) \wedge ((v4\_relat\_2\ X1) \wedge ((v6\_relat\_2\ X1) \wedge (( \\
& v8\_relat\_2\ X1) \wedge (m1\_subset\_1\ X1\ (k1\_zfmisc\_1\ (k2\_zfmisc\_1\ (k15\_pre\_poly \\
& \quad X0)\ (k15\_pre\_poly\ X0)))))))))) \Rightarrow (\forall X2.((\neg v7\_struct\_0\ X2) \wedge \\
& ((v13\_algstr\_0\ X2) \wedge ((v33\_algstr\_0\ X2) \wedge ((v3\_group\_1\ X2) \wedge ((v5\_group\_1 \\
& \quad X2) \wedge ((v4\_vectsp\_1\ X2) \wedge ((v5\_vectsp\_1\ X2) \wedge ((v3\_rlvect\_1\ X2) \wedge \\
& \quad ((v4\_rlvect\_1\ X2) \wedge (l6\_algstr\_0\ X2)))))))))) \Rightarrow (\forall X3.(( \\
& \quad v1\_funct\_1\ X3) \wedge ((v1\_funct\_2\ X3\ (k15\_pre\_poly\ X0)\ (u1\_struct\_0 \\
& \quad X2)) \wedge ((v1\_polynom1\ X3\ (k15\_pre\_poly\ X0)\ X2) \wedge (m1\_subset\_1\ X3\ ( \\
& \quad k1\_zfmisc\_1\ (k2\_zfmisc\_1\ (k15\_pre\_poly\ X0)\ (u1\_struct\_0\ X2)))))) \Rightarrow \\
& \quad (\forall X4.((v1\_funct\_1\ X4) \wedge ((v1\_funct\_2\ X4\ (k15\_pre\_poly\ X0) \\
& \quad (u1\_struct\_0\ X2)) \wedge ((v1\_polynom1\ X4\ (k15\_pre\_poly\ X0)\ X2) \wedge (m1\_subset\_1 \\
& \quad X4\ (k1\_zfmisc\_1\ (k2\_zfmisc\_1\ (k15\_pre\_poly\ X0)\ (u1\_struct\_0\ X2)))))) \Rightarrow \\
& \quad (\forall X5.((v1\_funct\_1\ X5) \wedge ((v1\_funct\_2\ X5\ (k15\_pre\_poly\ X0) \\
& \quad (u1\_struct\_0\ X2)) \wedge ((v1\_polynom1\ X5\ (k15\_pre\_poly\ X0)\ X2) \wedge (m1\_subset\_1 \\
& \quad X5\ (k1\_zfmisc\_1\ (k2\_zfmisc\_1\ (k15\_pre\_poly\ X0)\ (u1\_struct\_0\ X2)))))) \Rightarrow \\
& ((r4\_polyred\ X0\ X1\ X2\ X3\ X5\ X4) \Rightarrow ((X3 \neq k7\_polynom1\ X0\ X2) \wedge (X5 \neq k7\_polynom1 \\
& \quad X0\ X2))))))
\end{aligned}$$