

## l99\_interval

(TMPMnm32muFGg1YBsxgvz8zhzdX6tmeemmF)

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Let  $v2\_struct\_0 : \iota \Rightarrow o$  be given. Let  $v3\_roughs\_1 : \iota \Rightarrow o$  be given. Let  $l1\_orders\_2 : \iota \Rightarrow o$  be given. Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k18\_interval : \iota \Rightarrow \iota$  be given. Let  $k1\_binop\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $u2\_lattices : \iota \Rightarrow \iota$  be given. Let  $k21\_interval : \iota \Rightarrow \iota$  be given. Let  $u1\_lattices : \iota \Rightarrow \iota$  be given. Let  $m2\_interval : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $r2\_interval : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k16\_interval : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k17\_interval : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v1\_xboole\_0 : \iota \Rightarrow o$  be given. Let  $v3\_lattices : \iota \Rightarrow o$  be given. Let  $l3\_lattices : \iota \Rightarrow o$  be given. Let  $u1\_struct\_0 : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall X0. ((\neg v2\_struct\_0 X0) \wedge ((v3\_roughs\_1 X0) \wedge (l1\_orders\_2 \\ & X0))) \Rightarrow (\forall X1. (m2\_interval X1 X0) \Rightarrow (\forall X2. (m2\_interval \\ & X2 X0) \Rightarrow (r2\_interval X0 (k16\_interval X0 X1 (k17\_interval X0 X1 X2)) \\ & X1))) \end{aligned} \tag{1}$$

Assume the following.

$$\begin{aligned} & \forall X0. ((\neg v2\_struct\_0 X0) \wedge ((v3\_roughs\_1 X0) \wedge (l1\_orders\_2 \\ & X0))) \Rightarrow (\forall X1. (m2\_interval X1 X0) \Rightarrow (\forall X2. (m2\_interval \\ & X2 X0) \Rightarrow (r2\_interval X0 (k17\_interval X0 X1 X2) (k17\_interval X0 \\ & X2 X1)))) \end{aligned} \tag{2}$$

Assume the following.

$$\begin{aligned} & \forall X0. ((\neg v2\_struct\_0 X0) \wedge ((v3\_roughs\_1 X0) \wedge (l1\_orders\_2 \\ & X0))) \Rightarrow (\forall X1. (m2\_interval X1 X0) \Rightarrow (\forall X2. (m2\_interval \\ & X2 X0) \Rightarrow (r2\_interval X0 (k16\_interval X0 X1 X2) (k16\_interval X0 \\ & X2 X1)))) \end{aligned} \tag{3}$$

Assume the following.

$$\forall X0. \forall X1. (m1\_subset\_1 X0 X1) \Rightarrow ((v1\_xboole\_0 X1) \vee (X0 \in X1)) \tag{4}$$

Assume the following.

$$\forall X0. \forall X1. (X0 \in X1) \Rightarrow (m1\_subset\_1 X0 X1) \tag{5}$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge((v3\_roughs\_1 X0)\wedge(l1\_orders\_2 X0)))\wedge((m2\_interval1 X1 X0)\wedge(m2\_interval1 X2 X0)))\Rightarrow((r2\_interval1 X0 X1 X2)\Leftrightarrow(X1 = X2)) \quad (6)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0)\wedge((v3\_roughs\_1 X0)\wedge(l1\_orders\_2 X0)))\Rightarrow(\exists X1.m2\_interval1 X1 X0) \quad (7)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0)\wedge((v3\_roughs\_1 X0)\wedge(l1\_orders\_2 X0)))\Rightarrow((v3\_lattices (k21\_interval1 X0))\wedge(l3\_lattices (k21\_interval1 X0))) \quad (8)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge((v3\_roughs\_1 X0)\wedge(l1\_orders\_2 X0)))\wedge((m2\_interval1 X1 X0)\wedge(m2\_interval1 X2 X0)))\Rightarrow(m2\_interval1 (k17\_interval1 X0 X1 X2) X0) \quad (9)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge((v3\_roughs\_1 X0)\wedge(l1\_orders\_2 X0)))\wedge((m2\_interval1 X1 X0)\wedge(m2\_interval1 X2 X0)))\Rightarrow(m2\_interval1 (k16\_interval1 X0 X1 X2) X0) \quad (10)$$

Assume the following.

$$\begin{aligned} \forall X0.((\neg v2\_struct\_0 X0)\wedge((v3\_roughs\_1 X0)\wedge(l1\_orders\_2 X0)))\Rightarrow(\forall X1.((v3\_lattices X1)\wedge(l3\_lattices X1))\Rightarrow((X1 = k21\_interval1 X0)\Leftrightarrow((u1\_struct\_0 X1 = k18\_interval1 X0)\wedge(\forall X2. \\ (m1\_subset\_1 X2 (k18\_interval1 X0))\Rightarrow(\forall X3.(m1\_subset\_1 X3 (k18\_interval1 X0))\Rightarrow(\forall X4.(m2\_interval1 X4 X0)\Rightarrow(\forall X5. \\ (m2\_interval1 X5 X0)\Rightarrow(((X2 = X4)\wedge(X3 = X5))\Rightarrow((k1\_binop\_1 (u2\_lattices X1) X2 X3 = k16\_interval1 X0 X4 X5)\wedge(k1\_binop\_1 (u1\_lattices X1) X2 X3 = k17\_interval1 X0 X4 X5)))))))))) \end{aligned} \quad (11)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0)\wedge((v3\_roughs\_1 X0)\wedge(l1\_orders\_2 X0)))\Rightarrow(\forall X1.(X1 = k18\_interval1 X0)\Leftrightarrow(\forall X2.(X2 \in X1)\Leftrightarrow(m2\_interval1 X2 X0)) \quad (12)$$

Assume the following.

$$\forall X0.(v1\_xboole\_0 X0)\Leftrightarrow(\forall X1.\neg X1 \in X0) \quad (13)$$

**Theorem 1**

$$\begin{aligned} \forall X0.((\neg v2\_struct\_0 X0) \wedge (v3\_roughs\_1 X0) \wedge (l1\_orders\_2 \\ X0)) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (k18\_interval X0)) \Rightarrow (\forall X2. \\ (m1\_subset\_1 X2 (k18\_interval X0)) \Rightarrow (k1\_binop\_1 (u2\_lattices \\ (k21\_interval X0)) (k1\_binop\_1 (u1\_lattices (k21\_interval X0)) \\ X1 X2) X2 = X2))) \end{aligned}$$