

# t104\_pboole (TMRrAdXynRJe- ZoyH85mXLbZjAZBiZCK1QhA)

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Let  $v1\_relat\_1 : \iota \Rightarrow o$  be given. Let  $v4\_relat\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v1\_funct\_1 : \iota \Rightarrow o$  be given. Let  $v1\_partfun1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $r4\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k4\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $r2\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Assume the following.

$$\begin{aligned}
 & \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\
 & (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1\_relat\_1 \\
 & X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\
 & (\forall X3. ((v1\_relat\_1 X3) \wedge ((v4\_relat\_1 X3 X0) \wedge ((v1\_funct\_1 \\
 & X3) \wedge (v1\_partfun1 X3 X0)))) \Rightarrow ((r4\_pboole X0 X1 X2) \wedge (r2\_pboole \\
 & X0 X2 X3)) \Rightarrow (r4\_pboole X0 X1 X3)))
 \end{aligned} \tag{1}$$

Assume the following.

$$\begin{aligned}
 & \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\
 & (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1\_relat\_1 \\
 & X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\
 & (r2\_pboole X0 (k4\_pboole X0 X1 X2) X1))
 \end{aligned} \tag{2}$$

Assume the following.

$$\begin{aligned}
 & \forall X0. \forall X1. \forall X2. (((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 \\
 & X1 X0) \wedge ((v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0)))) \wedge ((v1\_relat\_1 \\
 & X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\
 & ((v1\_relat\_1 (k4\_pboole X0 X1 X2)) \wedge ((v4\_relat\_1 (k4\_pboole X0 \\
 & X1 X2) X0) \wedge ((v1\_funct\_1 (k4\_pboole X0 X1 X2)) \wedge (v1\_partfun1 (k4\_pboole \\
 & X0 X1 X2) X0))))
 \end{aligned} \tag{3}$$

**Theorem 1**

$$\begin{aligned} & \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\ & (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1\_relat\_1 \\ & X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\ & (\forall X3. ((v1\_relat\_1 X3) \wedge ((v4\_relat\_1 X3 X0) \wedge ((v1\_funct\_1 \\ & X3) \wedge (v1\_partfun1 X3 X0)))) \Rightarrow ((r4\_pboole X0 X1 (k4\_pboole X0 X2 X3)) \Rightarrow \\ & (r4\_pboole X0 X1 X2))) \end{aligned}$$