

# t106\_pboole (TMRkb- NXkaq1gRUfBvJdKebNhg7H9gKwv9BX)

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Let  $v1\_relat\_1 : \iota \Rightarrow o$  be given. Let  $v4\_relat\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v1\_funct\_1 : \iota \Rightarrow o$  be given. Let  $v1\_partfun1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $r4\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k4\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $r1\_xboole\_0 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k4\_xboole\_0 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k6\_subset\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_funct\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall X0. \forall X1. \forall X2. (r1\_xboole\_0 X0 (k4\_xboole\_0 X1 X2)) \Rightarrow (r1\_xboole\_0 X1 (k4\_xboole\_0 X0 X2)) \quad (1)$$

Assume the following.

$$\forall X0. \forall X1. k6\_subset\_1 X0 X1 = k4\_xboole\_0 X0 X1 \quad (2)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 \\ & X1 X0) \wedge ((v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0)))) \wedge ((v1\_relat\_1 \\ & X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))))) \Rightarrow \\ & ((v1\_relat\_1 (k4\_pboole X0 X1 X2)) \wedge ((v4\_relat\_1 (k4\_pboole X0 \\ & X1 X2) X0) \wedge ((v1\_funct\_1 (k4\_pboole X0 X1 X2)) \wedge (v1\_partfun1 (k4\_pboole \\ & X0 X1 X2) X0)))) \end{aligned} \quad (3)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\ & (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1\_relat\_1 \\ & X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\ & ((r4\_pboole X0 X1 X2) \Leftrightarrow (\forall X3. \neg (X3 \in X0) \wedge (r1\_xboole\_0 (k1\_funct\_1 \\ & X1 X3) (k1\_funct\_1 X2 X3)))) \end{aligned} \quad (4)$$

Assume the following.

$$\begin{aligned}
& \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\
& (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1\_relat\_1 \\
& X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\
& (\forall X3. ((v1\_relat\_1 X3) \wedge ((v4\_relat\_1 X3 X0) \wedge ((v1\_funct\_1 \\
& X3) \wedge (v1\_partfun1 X3 X0)))) \Rightarrow ((X3 = k4\_pboole X0 X1 X2) \Leftrightarrow (\forall X4. \\
& (X4 \in X0) \Rightarrow (k1\_funct\_1 X3 X4 = k6\_subset\_1 (k1\_funct\_1 X1 X4) (k1\_funct\_1 \\
& X2 X4))))))
\end{aligned} \tag{5}$$

**Theorem 1**

$$\begin{aligned}
& \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\
& (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1\_relat\_1 \\
& X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\
& (\forall X3. ((v1\_relat\_1 X3) \wedge ((v4\_relat\_1 X3 X0) \wedge ((v1\_funct\_1 \\
& X3) \wedge (v1\_partfun1 X3 X0)))) \Rightarrow ((r4\_pboole X0 X1 (k4\_pboole X0 X2 X3)) \Rightarrow \\
& (r4\_pboole X0 X2 (k4\_pboole X0 X1 X3))))
\end{aligned}$$