

t108_pboole (TMNTyvuumzmvjXYfJKmDiy- VUx7XRjUyaAa)

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Let $v1_relat_1 : \iota \Rightarrow o$ be given. Let $v4_relat_1 : \iota \Rightarrow \iota \Rightarrow o$ be given. Let $v1_funct_1 : \iota \Rightarrow o$ be given. Let $v1_partfun1 : \iota \Rightarrow \iota \Rightarrow o$ be given. Let $r5_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$ be given. Let $k4_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Let $r1_xboole_0 : \iota \Rightarrow \iota \Rightarrow o$ be given. Let $k4_xboole_0 : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Let $k6_subset_1 : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Let $k1_funct_1 : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall X0. \forall X1. r1_xboole_0 (k4_xboole_0 X0 X1) X1 \quad (1)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((v1_relat_1 X1) \wedge ((v4_relat_1 \\ & X1 X0) \wedge ((v1_funct_1 X1) \wedge (v1_partfun1 X1 X0)))) \wedge ((v1_relat_1 \\ & X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))))) \Rightarrow \\ & (r5_pboole X0 X1 X2) \Rightarrow (r5_pboole X0 X2 X1) \end{aligned} \quad (2)$$

Assume the following.

$$\forall X0. \forall X1. k6_subset_1 X0 X1 = k4_xboole_0 X0 X1 \quad (3)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((v1_relat_1 X1) \wedge ((v4_relat_1 \\ & X1 X0) \wedge ((v1_funct_1 X1) \wedge (v1_partfun1 X1 X0)))) \wedge ((v1_relat_1 \\ & X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))))) \Rightarrow \\ & ((v1_relat_1 (k4_pboole X0 X1 X2)) \wedge ((v4_relat_1 (k4_pboole X0 \\ & X1 X2) X0) \wedge ((v1_funct_1 (k4_pboole X0 X1 X2)) \wedge (v1_partfun1 (k4_pboole \\ & X0 X1 X2) X0)))) \end{aligned} \quad (4)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((v1_relat_1 X1) \wedge ((v4_relat_1 X1 X0) \wedge \\ & (v1_funct_1 X1) \wedge (v1_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1_relat_1 \\ & X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))) \Rightarrow \\ & ((r5_pboole X0 X1 X2) \Leftrightarrow (\forall X3. (X3 \in X0) \Rightarrow (r1_xboole_0 (k1_funct_1 \\ & X1 X3) (k1_funct_1 X2 X3)))) \end{aligned} \quad (5)$$

Assume the following.

$$\begin{aligned}
& \forall X0. \forall X1. ((v1_relat_1 X1) \wedge ((v4_relat_1 X1 X0) \wedge \\
& (v1_funct_1 X1) \wedge (v1_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1_relat_1 \\
& X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))) \Rightarrow \\
& (\forall X3. ((v1_relat_1 X3) \wedge ((v4_relat_1 X3 X0) \wedge ((v1_funct_1 \\
& X3) \wedge (v1_partfun1 X3 X0)))) \Rightarrow ((X3 = k4_pboole X0 X1 X2) \Leftrightarrow (\forall X4. \\
& (X4 \in X0) \Rightarrow (k1_funct_1 X3 X4 = k6_subset_1 (k1_funct_1 X1 X4) (k1_funct_1 \\
& X2 X4))))))
\end{aligned} \tag{6}$$

Theorem 1

$$\begin{aligned}
& \forall X0. \forall X1. ((v1_relat_1 X1) \wedge ((v4_relat_1 X1 X0) \wedge \\
& (v1_funct_1 X1) \wedge (v1_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1_relat_1 \\
& X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))) \Rightarrow \\
& (r5_pboole X0 X1 (k4_pboole X0 X2 X1)))
\end{aligned}$$