

## t10\_closure1

(TMcRPkmwHS84bxQB8CcVebrfyQ5rq6wusLZ)

October 27, 2020

Let  $v1\_relat\_1 : \iota \Rightarrow o$  be given. Let  $v2\_relat\_1 : \iota \Rightarrow o$  be given. Let  $v4\_relat\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v1\_funct\_1 : \iota \Rightarrow o$  be given. Let  $v1\_partfun1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $m1\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k2\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $r6\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k16\_pralg\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k2\_msualg\_3 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_closure1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $m2\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k15\_pralg\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v1\_funcop\_1 : \iota \Rightarrow o$  be given. Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v2\_relat\_1 X1) \wedge ((v4\_relat\_1 \\ & X1 X0) \wedge ((v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0)))))) \Rightarrow (\forall X2. \\ (m1\_pboole X2 X0 X1) \Rightarrow (r6\_pboole X0 X2 (k1\_closure1 X0 X1 X1 (k2\_msualg\_3 \\ X0 X1) X2))) \end{aligned} \tag{1}$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 \\ X1 X0) \wedge ((v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0)))) \wedge ((v1\_relat\_1 \\ X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))))) \Rightarrow \\ & ((r6\_pboole X0 X1 X2) \Leftrightarrow (X1 = X2)) \end{aligned} \tag{2}$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. \forall X3. \forall X4. (((v1\_relat\_1 \\ X1) \wedge ((v2\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge ((v1\_funct\_1 X1) \wedge ( \\ v1\_partfun1 X1 X0)))))) \wedge (((v1\_relat\_1 X2) \wedge ((v2\_relat\_1 X2) \wedge ( \\ v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))))) \wedge \\ & ((m2\_pboole X3 X0 X1 X2) \wedge (m1\_pboole X4 X0 X1))) \Rightarrow (k1\_closure1 X0 \\ X1 X2 X3 X4 = k15\_pralg\_1 X3 X4) \end{aligned} \tag{3}$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(((v1\_relat\_1 X1)\wedge((v4\_relat\_1 \\ & X1 X0)\wedge((v1\_funct\_1 X1)\wedge((v1\_partfun1 X1 X0)\wedge(v1\_funcop\_1 X1))))\wedge \\ & ((v1\_relat\_1 X2)\wedge((v4\_relat\_1 X2 X0)\wedge((v1\_funct\_1 X2)\wedge(v1\_partfun1 \\ & X2 X0))))\Rightarrow(k16\_pralg\_1 X0 X1 X2 = k15\_pralg\_1 X1 X2) \end{aligned} \quad (4)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(((v1\_relat\_1 X1)\wedge((v4\_relat\_1 \\ & X1 X0)\wedge((v1\_funct\_1 X1)\wedge(v1\_partfun1 X1 X0))))\wedge((v1\_relat\_1 \\ & X2)\wedge((v4\_relat\_1 X2 X0)\wedge((v1\_funct\_1 X2)\wedge(v1\_partfun1 X2 X0))))\Rightarrow \\ & (\forall X3.(m2\_pboole X3 X0 X1 X2)\Rightarrow((v1\_relat\_1 X3)\wedge((v4\_relat\_1 \\ & X3 X0)\wedge((v1\_funct\_1 X3)\wedge(v1\_partfun1 X3 X0)))) \end{aligned} \quad (5)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.((v1\_relat\_1 X1)\wedge((v4\_relat\_1 X1 X0)\wedge( \\ & (v1\_funct\_1 X1)\wedge(v1\_partfun1 X1 X0))))\Rightarrow(\forall X2.(m1\_pboole \\ & X2 X0 X1)\Rightarrow((v1\_relat\_1 X2)\wedge((v4\_relat\_1 X2 X0)\wedge((v1\_funct\_1 X2)\wedge \\ & (v1\_partfun1 X2 X0)))) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.((v1\_relat\_1 X1)\wedge((v4\_relat\_1 X1 X0)\wedge( \\ & (v1\_funct\_1 X1)\wedge(v1\_partfun1 X1 X0))))\Rightarrow(m2\_pboole (k2\_msualg\_3 \\ & X0 X1) X0 X1 X1) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.\forall X3.\forall X4.(((v1\_relat\_1 \\ & X1)\wedge((v2\_relat\_1 X1)\wedge((v4\_relat\_1 X1 X0)\wedge((v1\_funct\_1 X1)\wedge( \\ & v1\_partfun1 X1 X0))))\wedge((v1\_relat\_1 X2)\wedge((v2\_relat\_1 X2)\wedge( \\ & (v4\_relat\_1 X2 X0)\wedge((v1\_funct\_1 X2)\wedge(v1\_partfun1 X2 X0))))\wedge \\ & ((m2\_pboole X3 X0 X1 X2)\wedge(m1\_pboole X4 X0 X1)))\Rightarrow(m1\_pboole (k1\_closure1 \\ & X0 X1 X2 X3 X4) X0 X2) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(((v1\_relat\_1 X1)\wedge((v4\_relat\_1 \\ & X1 X0)\wedge((v1\_funct\_1 X1)\wedge(v1\_partfun1 X1 X0))))\wedge((v1\_relat\_1 \\ & X2)\wedge((v4\_relat\_1 X2 X0)\wedge((v1\_funct\_1 X2)\wedge(v1\_partfun1 X2 X0))))\Rightarrow \\ & (\forall X3.(m2\_pboole X3 X0 X1 X2)\Rightarrow(v1\_funcop\_1 X3)) \end{aligned} \quad (9)$$

**Theorem 1**

$$\begin{aligned} & \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v2\_relat\_1 X1) \wedge ((v4\_relat\_1 \\ & X1 X0) \wedge ((v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0)))))) \Rightarrow (\forall X2. \\ & (m1\_pboole X2 X0 X1) \Rightarrow (\forall X3. (m1\_pboole X3 X0 X1) \Rightarrow ((m1\_pboole \\ & (k2\_pboole X0 X2 X3) X0 X1) \Rightarrow (r6\_pboole X0 (k16\_pralg\_1 X0 (k2\_msualg\_3 \\ & X0 X1) (k2\_pboole X0 X2 X3)) (k2\_pboole X0 (k1\_closure1 X0 X1 X1 (k2\_msualg\_3 \\ & X0 X1) X2) (k1\_closure1 X0 X1 X1 (k2\_msualg\_3 X0 X1) X3)))))) \end{aligned}$$