

t10\_closure2  
(TMKaT5YSbdA7NGQaTkLYnAJAmjsvVZiHCfS)

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Let  $v1\_relat\_1 : \iota \Rightarrow o$  be given. Let  $v4\_relat\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v1\_funct\_1 : \iota \Rightarrow o$  be given. Let  $v1\_partfun1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $m1\_closure2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k2\_closure2 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k2\_pboole : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $r2\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k1\_closure2 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v1\_xboole\_0 : \iota \Rightarrow o$  be given. Let  $v4\_funct\_1 : \iota \Rightarrow o$  be given. Let  $v2\_card\_3 : \iota \Rightarrow o$  be given. Let  $m3\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k1\_zfmisc\_1 : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\ & (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1\_relat\_1 \\ & X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\ & (\forall X3. ((v1\_relat\_1 X3) \wedge ((v4\_relat\_1 X3 X0) \wedge ((v1\_funct\_1 \\ & X3) \wedge (v1\_partfun1 X3 X0)))) \Rightarrow (\forall X4. ((v1\_relat\_1 X4) \wedge ((v4\_relat\_1 \\ & X4 X0) \wedge ((v1\_funct\_1 X4) \wedge (v1\_partfun1 X4 X0)))) \Rightarrow (((r2\_pboole \\ & X0 X1 X2) \wedge (r2\_pboole X0 X3 X4)) \Rightarrow (r2\_pboole X0 (k2\_pboole X0 X1 X3) \\ & (k2\_pboole X0 X2 X4)))))) \end{aligned} \tag{1}$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\ & (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (k2\_closure2 X0 X1 = k1\_closure2 \\ & X0 X1) \end{aligned} \tag{2}$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 \\ & X1 X0) \wedge ((v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0)))) \wedge ((v1\_relat\_1 \\ & X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\ & (k2\_pboole X0 X1 X1 = X1) \end{aligned} \tag{3}$$

Assume the following.

$$\forall X0.\forall X1.((v1\_relat\_1 X1)\wedge((v4\_relat\_1 X1 X0)\wedge(v1\_funct\_1 X1)\wedge(v1\_partfun1 X1 X0)))\Rightarrow((\neg v1\_xboole\_0 (k1\_closure2 X0 X1))\wedge((v4\_funct\_1 (k1\_closure2 X0 X1))\wedge(v2\_card\_3 (k1\_closure2 X0 X1)))) \quad (4)$$

Assume the following.

$$\forall X0.\forall X1.((v1\_relat\_1 X1)\wedge((v4\_relat\_1 X1 X0)\wedge(v1\_funct\_1 X1)\wedge(v1\_partfun1 X1 X0)))\Rightarrow(\forall X2.(m3\_pboole X2 X0 X1)\Rightarrow((v1\_relat\_1 X2)\wedge((v4\_relat\_1 X2 X0)\wedge((v1\_funct\_1 X2)\wedge(v1\_partfun1 X2 X0)))))) \quad (5)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((v1\_relat\_1 X1)\wedge((v4\_relat\_1 X1 X0)\wedge((v1\_funct\_1 X1)\wedge(v1\_partfun1 X1 X0))))\wedge((\neg v1\_xboole\_0 X2)\wedge(m1\_subset\_1 X2 (k1\_zfmisc\_1 (k1\_closure2 X0 X1))))))\Rightarrow(\forall X3.(m1\_closure2 X3 X0 X1 X2)\Rightarrow(m3\_pboole X3 X0 X1)) \quad (6)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((v1\_relat\_1 X1)\wedge((v4\_relat\_1 X1 X0)\wedge((v1\_funct\_1 X1)\wedge(v1\_partfun1 X1 X0))))\wedge((v1\_relat\_1 X2)\wedge((v4\_relat\_1 X2 X0)\wedge((v1\_funct\_1 X2)\wedge(v1\_partfun1 X2 X0))))))\Rightarrow((v1\_relat\_1 (k2\_pboole X0 X1 X2))\wedge((v4\_relat\_1 (k2\_pboole X0 X1 X2) X0)\wedge((v1\_funct\_1 (k2\_pboole X0 X1 X2))\wedge(v1\_partfun1 (k2\_pboole X0 X1 X2) X0)))))) \quad (7)$$

Assume the following.

$$\forall X0.\forall X1.((v1\_relat\_1 X1)\wedge((v4\_relat\_1 X1 X0)\wedge(v1\_funct\_1 X1)\wedge(v1\_partfun1 X1 X0)))\Rightarrow(m1\_subset\_1 (k2\_closure2 X0 X1) (k1\_zfmisc\_1 (k1\_closure2 X0 X1))) \quad (8)$$

Assume the following.

$$\forall X0.\forall X1.((v1\_relat\_1 X1)\wedge((v4\_relat\_1 X1 X0)\wedge(v1\_funct\_1 X1)\wedge(v1\_partfun1 X1 X0)))\Rightarrow(\forall X2.(X2 = k1\_closure2 X0 X1)\Leftrightarrow(\forall X3.(X3 \in X2)\Leftrightarrow(m3\_pboole X3 X0 X1))) \quad (9)$$

Assume the following.

$$\forall X0.\forall X1.((v1\_relat\_1 X1)\wedge((v4\_relat\_1 X1 X0)\wedge(v1\_funct\_1 X1)\wedge(v1\_partfun1 X1 X0)))\Rightarrow(\forall X2.((v1\_relat\_1 X2)\wedge((v4\_relat\_1 X2 X0)\wedge((v1\_funct\_1 X2)\wedge(v1\_partfun1 X2 X0))))\Rightarrow((m3\_pboole X2 X0 X1)\Leftrightarrow(r2\_pboole X0 X2 X1))) \quad (10)$$

**Theorem 1**

$$\begin{aligned} & \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\ & (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (\forall X2. (m1\_closure2 \\ & X2 X0 X1 (k2\_closure2 X0 X1)) \Rightarrow (\forall X3. (m1\_closure2 X3 X0 X1 ( \\ & k2\_closure2 X0 X1)) \Rightarrow (k2\_pboole X0 X2 X3 \in k2\_closure2 X0 X1))) \end{aligned}$$