

# t110\_pboole (TMTtL- bqxXDYyR9VrUpfB43FMXMvAqQUyust)

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Let  $v1\_relat\_1 : \iota \Rightarrow o$  be given. Let  $v4\_relat\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v1\_funct\_1 : \iota \Rightarrow o$  be given. Let  $v1\_partfun1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $r5\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k3\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k5\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_funct\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k5\_xboole\_0 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $r1\_xboole\_0 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k3\_xboole\_0 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\ & (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1\_relat\_1 \\ & X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\ & (\forall X3. (X3 \in X0) \Rightarrow (k1\_funct\_1 (k5\_pboole X0 X1 X2) X3 = k5\_xboole\_0 \\ & (k1\_funct\_1 X1 X3) (k1\_funct\_1 X2 X3)))) \end{aligned} \quad (1)$$

Assume the following.

$$\forall X0. \forall X1. r1\_xboole\_0 (k3\_xboole\_0 X0 X1) (k5\_xboole\_0 X0 X1) \quad (2)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 \\ & X1 X0) \wedge ((v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0)))) \wedge ((v1\_relat\_1 \\ & X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\ & ((v1\_relat\_1 (k5\_pboole X0 X1 X2)) \wedge ((v4\_relat\_1 (k5\_pboole X0 \\ & X1 X2) X0) \wedge ((v1\_funct\_1 (k5\_pboole X0 X1 X2)) \wedge (v1\_partfun1 (k5\_pboole \\ & X0 X1 X2) X0)))) \end{aligned} \quad (3)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 \\ & X1 X0) \wedge ((v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0)))) \wedge ((v1\_relat\_1 \\ & X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\ & ((v1\_relat\_1 (k3\_pboole X0 X1 X2)) \wedge ((v4\_relat\_1 (k3\_pboole X0 \\ & X1 X2) X0) \wedge ((v1\_funct\_1 (k3\_pboole X0 X1 X2)) \wedge (v1\_partfun1 (k3\_pboole \\ & X0 X1 X2) X0)))) \end{aligned} \quad (4)$$

Assume the following.

$$\begin{aligned}
& \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\
& (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1\_relat\_1 \\
& X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\
& ((r5\_pboole X0 X1 X2) \Leftrightarrow (\forall X3. (X3 \in X0) \Rightarrow (r1\_xboole\_0 (k1\_funct\_1 \\
& X1 X3) (k1\_funct\_1 X2 X3))))))
\end{aligned} \tag{5}$$

Assume the following.

$$\begin{aligned}
& \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\
& (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1\_relat\_1 \\
& X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\
& (\forall X3. ((v1\_relat\_1 X3) \wedge ((v4\_relat\_1 X3 X0) \wedge ((v1\_funct\_1 \\
& X3) \wedge (v1\_partfun1 X3 X0)))) \Rightarrow ((X3 = k3\_pboole X0 X1 X2) \Leftrightarrow (\forall X4. \\
& (X4 \in X0) \Rightarrow (k1\_funct\_1 X3 X4 = k3\_xboole\_0 (k1\_funct\_1 X1 X4) (k1\_funct\_1 \\
& X2 X4))))))
\end{aligned} \tag{6}$$

**Theorem 1**

$$\begin{aligned}
& \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\
& (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1\_relat\_1 \\
& X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\
& (r5\_pboole X0 (k3\_pboole X0 X1 X2) (k5\_pboole X0 X1 X2)))
\end{aligned}$$