

# t11\_lattices (TM- TAm4csVdchGjSS4rJGAKRP77RKqDtMr24)

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Let  $v2\_struct\_0 : \iota \Rightarrow o$  be given. Let  $v10\_lattices : \iota \Rightarrow o$  be given. Let  $v11\_lattices : \iota \Rightarrow o$  be given. Let  $l3\_lattices : \iota \Rightarrow o$  be given. Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $u1\_struct\_0 : \iota \Rightarrow \iota$  be given. Let  $k3\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k4\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v6\_lattices : \iota \Rightarrow o$  be given. Let  $l1\_lattices : \iota \Rightarrow o$  be given. Let  $k2\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v4\_lattices : \iota \Rightarrow o$  be given. Let  $l2\_lattices : \iota \Rightarrow o$  be given. Let  $k1\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v8\_lattices : \iota \Rightarrow o$  be given. Let  $v9\_lattices : \iota \Rightarrow o$  be given. Let  $v5\_lattices : \iota \Rightarrow o$  be given. Let  $v7\_lattices : \iota \Rightarrow o$  be given. Assume the following.

$$\begin{aligned} \forall X0. \forall X1. \forall X2. (((\neg v2\_struct\_0 X0) \wedge (v6\_lattices \\ X0) \wedge (l1\_lattices X0))) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge ( \\ m1\_subset\_1 X2 (u1\_struct\_0 X0))) \Rightarrow (k4\_lattices X0 X1 X2 = k2\_lattices \\ X0 X1 X2) \end{aligned} \tag{1}$$

Assume the following.

$$\begin{aligned} \forall X0. \forall X1. \forall X2. (((\neg v2\_struct\_0 X0) \wedge (v4\_lattices \\ X0) \wedge (l2\_lattices X0))) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge ( \\ m1\_subset\_1 X2 (u1\_struct\_0 X0))) \Rightarrow (k3\_lattices X0 X1 X2 = k1\_lattices \\ X0 X1 X2) \end{aligned} \tag{2}$$

Assume the following.

$$\begin{aligned} \forall X0. \forall X1. (((\neg v2\_struct\_0 X0) \wedge (v6\_lattices X0) \wedge \\ ((v8\_lattices X0) \wedge (v9\_lattices X0) \wedge (l3\_lattices X0)))) \wedge ( \\ m1\_subset\_1 X1 (u1\_struct\_0 X0)) \Rightarrow (k4\_lattices X0 X1 X1 = X1) \end{aligned} \tag{3}$$

Assume the following.

$$\forall X0. (l3\_lattices X0) \Rightarrow ((l1\_lattices X0) \wedge (l2\_lattices X0)) \tag{4}$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge((v4\_lattices X0)\wedge(l2\_lattices X0)))\wedge((m1\_subset\_1 X1 (u1\_struct\_0 X0))\wedge(m1\_subset\_1 X2 (u1\_struct\_0 X0))))\Rightarrow(m1\_subset\_1 (k3\_lattices X0 X1 X2) (u1\_struct\_0 X0)) \quad (5)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge(l1\_lattices X0))\wedge((m1\_subset\_1 X1 (u1\_struct\_0 X0))\wedge(m1\_subset\_1 X2 (u1\_struct\_0 X0))))\Rightarrow(m1\_subset\_1 (k2\_lattices X0 X1 X2) (u1\_struct\_0 X0)) \quad (6)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0)\wedge(l3\_lattices X0))\Rightarrow((v8\_lattices X0)\Leftrightarrow(\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 X0))\Rightarrow(\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 X0))\Rightarrow(k1\_lattices X0 (k2\_lattices X0 X1 X2) X2 = X2)))) \quad (7)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0)\wedge(l2\_lattices X0))\Rightarrow((v5\_lattices X0)\Leftrightarrow(\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 X0))\Rightarrow(\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 X0))\Rightarrow(\forall X3.(m1\_subset\_1 X3 (u1\_struct\_0 X0))\Rightarrow(k1\_lattices X0 X1 (k1\_lattices X0 X2 X3) = k1\_lattices X0 (k1\_lattices X0 X1 X2) X3)))))) \quad (8)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0)\wedge(l3\_lattices X0))\Rightarrow((v11\_lattices X0)\Leftrightarrow(\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 X0))\Rightarrow(\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 X0))\Rightarrow(\forall X3.(m1\_subset\_1 X3 (u1\_struct\_0 X0))\Rightarrow(k2\_lattices X0 X1 (k1\_lattices X0 X2 X3) = k1\_lattices X0 (k2\_lattices X0 X1 X2) (k2\_lattices X0 X1 X3)))))) \quad (9)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0)\wedge(l3\_lattices X0))\Rightarrow((v10\_lattices X0)\Leftrightarrow(((v4\_lattices X0)\wedge((v5\_lattices X0)\wedge((v8\_lattices X0)\wedge((v6\_lattices X0)\wedge((v7\_lattices X0)\wedge(v9\_lattices X0)))))))) \quad (10)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge((v6\_lattices X0)\wedge(l1\_lattices X0)))\wedge((m1\_subset\_1 X1 (u1\_struct\_0 X0))\wedge(m1\_subset\_1 X2 (u1\_struct\_0 X0))))\Rightarrow(k4\_lattices X0 X1 X2 = k4\_lattices X0 X2 X1) \quad (11)$$

Assume the following.

$$\begin{aligned} \forall X0. \forall X1. \forall X2. (&((\neg v2\_struct\_0 X0) \wedge (v4\_lattices \\ &X0) \wedge (l2\_lattices X0))) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge \\ &m1\_subset\_1 X2 (u1\_struct\_0 X0))) \Rightarrow (k3\_lattices X0 X1 X2 = k3\_lattices \\ &X0 X2 X1) \end{aligned} \tag{12}$$

**Theorem 1**

$$\begin{aligned} \forall X0. (&(\neg v2\_struct\_0 X0) \wedge (v10\_lattices X0) \wedge (v11\_lattices \\ &X0) \wedge (l3\_lattices X0))) \Rightarrow (\forall X1. (m1\_subset\_1 X1 (u1\_struct\_0 \\ &X0)) \Rightarrow (\forall X2. (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (\forall X3. \\ &(m1\_subset\_1 X3 (u1\_struct\_0 X0)) \Rightarrow (k3\_lattices X0 X1 (k4\_lattices \\ &X0 X2 X3) = k4\_lattices X0 (k3\_lattices X0 X1 X2) (k3\_lattices X0 X1 \\ &X3)))))) \end{aligned}$$