

t11\_nat\_lat  
(TMR1YExW2zxFMioiB2tJTJkkA5srx4JhFt9)

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Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $u1\_struct\_0 : \iota \Rightarrow \iota$  be given. Let  $k3\_nat\_lat : \iota$  be given. Let  $k1\_binop\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_nat\_lat : \iota$  be given. Let  $k2\_nat\_lat : \iota$  be given. Let  $v7\_ordinal1 : \iota \Rightarrow o$  be given. Let  $k6\_nat\_d : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k5\_nat\_d : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k5\_numbers : \iota$  be given. Let  $k4\_ordinal1 : \iota$  be given. Let  $v1\_funct\_1 : \iota \Rightarrow o$  be given. Let  $v1\_funct\_2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k2\_zfmisc\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_zfmisc\_1 : \iota \Rightarrow \iota$  be given. Let  $k5\_binop\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k2\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $g3\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v6\_membered : \iota \Rightarrow o$  be given. Let  $l3\_lattices : \iota \Rightarrow o$  be given. Let  $l1\_lattices : \iota \Rightarrow o$  be given. Let  $l2\_lattices : \iota \Rightarrow o$  be given. Let  $v2\_struct\_0 : \iota \Rightarrow o$  be given. Let  $v3\_lattices : \iota \Rightarrow o$  be given. Let  $u1\_lattices : \iota \Rightarrow \iota$  be given. Let  $u2\_lattices : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall X0.(m1\_subset\_1 X0 (u1\_struct\_0 k3\_nat\_lat)) \Rightarrow (\forall X1. \\ & (m1\_subset\_1 X1 (u1\_struct\_0 k3\_nat\_lat)) \Rightarrow (k1\_binop\_1 k2\_nat\_lat \\ & X0 X1 = k1\_binop\_1 k2\_nat\_lat X1 X0)) \end{aligned} \quad (1)$$

Assume the following.

$$\forall X0.(v7\_ordinal1 X0) \Rightarrow (\forall X1.(v7\_ordinal1 X1) \Rightarrow (k6\_nat\_d X0 (k5\_nat\_d X0 X1) = X0)) \quad (2)$$

Assume the following.

$$k5\_numbers = k4\_ordinal1 \quad (3)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.\forall X3.(((v1\_funct\_1 X1) \wedge \\ & ((v1\_funct\_2 X1 (k2\_zfmisc\_1 X0 X0) X0) \wedge (m1\_subset\_1 X1 (k1\_zfmisc\_1 \\ & (k2\_zfmisc\_1 (k2\_zfmisc\_1 X0 X0) X0)))))) \wedge ((m1\_subset\_1 X2 X0) \wedge \\ & (m1\_subset\_1 X3 X0))) \Rightarrow (k5\_binop\_1 X0 X1 X2 X3 = k1\_binop\_1 X1 X2 X3) \end{aligned} \quad (4)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.((m1\_subset\_1 X0 (u1\_struct\_0 k3\_nat\_lat))\wedge \\ & (m1\_subset\_1 X1 (u1\_struct\_0 k3\_nat\_lat)))\Rightarrow(k2\_lattices k3\_nat\_lat \\ & X0 X1 = k6\_nat\_d X0 X1) \end{aligned} \quad (5)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.((m1\_subset\_1 X0 (u1\_struct\_0 k3\_nat\_lat))\wedge \\ & (m1\_subset\_1 X1 (u1\_struct\_0 k3\_nat\_lat)))\Rightarrow(k1\_lattices k3\_nat\_lat \\ & X0 X1 = k5\_nat\_d X0 X1) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(((v1\_funct\_1 X1)\wedge((v1\_funct\_2 \\ & X1 (k2\_zfmisc\_1 X0 X0) X0)\wedge(m1\_subset\_1 X1 (k1\_zfmisc\_1 (k2\_zfmisc\_1 \\ & (k2\_zfmisc\_1 X0 X0) X0))))\wedge((v1\_funct\_1 X2)\wedge((v1\_funct\_2 X2 \\ & (k2\_zfmisc\_1 X0 X0) X0)\wedge(m1\_subset\_1 X2 (k1\_zfmisc\_1 (k2\_zfmisc\_1 \\ & (k2\_zfmisc\_1 X0 X0) X0))))))\Rightarrow(\forall X3.\forall X4.\forall X5. \\ & (g3\_lattices X0 X1 X2 = g3\_lattices X3 X4 X5)\Rightarrow((X0 = X3)\wedge((X1 = X4)\wedge \\ & (X2 = X5)))) \end{aligned} \quad (7)$$

Assume the following.

$$v6\_membered k4\_ordinal1 \quad (8)$$

Assume the following.

$$v6\_membered (u1\_struct\_0 k3\_nat\_lat) \quad (9)$$

Assume the following.

$$\forall X0.(l3\_lattices X0)\Rightarrow((l1\_lattices X0)\wedge(l2\_lattices X0)) \quad (10)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.\forall X3.(((v1\_funct\_1 X1)\wedge \\ & ((v1\_funct\_2 X1 (k2\_zfmisc\_1 X0 X0) X0)\wedge(m1\_subset\_1 X1 (k1\_zfmisc\_1 \\ & (k2\_zfmisc\_1 (k2\_zfmisc\_1 X0 X0) X0))))\wedge((m1\_subset\_1 X2 X0)\wedge \\ & (m1\_subset\_1 X3 X0)))\Rightarrow(m1\_subset\_1 (k5\_binop\_1 X0 X1 X2 X3) X0) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} & (\neg v2\_struct\_0 k3\_nat\_lat)\wedge((v3\_lattices k3\_nat\_lat)\wedge(l3\_lattices \\ & k3\_nat\_lat)) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} & (v1\_funct\_1 k2\_nat\_lat)\wedge((v1\_funct\_2 k2\_nat\_lat (k2\_zfmisc\_1 \\ & k5\_numbers k5\_numbers) k5\_numbers)\wedge(m1\_subset\_1 k2\_nat\_lat \\ & (k1\_zfmisc\_1 (k2\_zfmisc\_1 (k2\_zfmisc\_1 k5\_numbers k5\_numbers) \\ & k5\_numbers)))) \end{aligned} \quad (13)$$

Assume the following.

$$(v1\_funct\_1\ k1\_nat\_lat) \wedge ((v1\_funct\_2\ k1\_nat\_lat\ (k2\_zfmisc\_1\ k5\_numbers\ k5\_numbers)\ k5\_numbers) \wedge (m1\_subset\_1\ k1\_nat\_lat\ (k1\_zfmisc\_1\ (k2\_zfmisc\_1\ (k2\_zfmisc\_1\ k5\_numbers\ k5\_numbers)\ k5\_numbers)))) \quad (14)$$

Assume the following.

$$k3\_nat\_lat = g3\_lattices\ k5\_numbers\ k2\_nat\_lat\ k1\_nat\_lat \quad (15)$$

Assume the following.

$$\forall X0. ((\neg v2\_struct\_0\ X0) \wedge (l1\_lattices\ X0)) \Rightarrow (\forall X1. (m1\_subset\_1\ X1\ (u1\_struct\_0\ X0)) \Rightarrow (\forall X2. (m1\_subset\_1\ X2\ (u1\_struct\_0\ X0)) \Rightarrow (k2\_lattices\ X0\ X1\ X2 = k5\_binop\_1\ (u1\_struct\_0\ X0)\ (u1\_lattices\ X0)\ X1\ X2)))) \quad (16)$$

Assume the following.

$$\forall X0. ((\neg v2\_struct\_0\ X0) \wedge (l2\_lattices\ X0)) \Rightarrow (\forall X1. (m1\_subset\_1\ X1\ (u1\_struct\_0\ X0)) \Rightarrow (\forall X2. (m1\_subset\_1\ X2\ (u1\_struct\_0\ X0)) \Rightarrow (k1\_lattices\ X0\ X1\ X2 = k5\_binop\_1\ (u1\_struct\_0\ X0)\ (u2\_lattices\ X0)\ X1\ X2)))) \quad (17)$$

Assume the following.

$$\forall X0. \forall X1. ((v7\_ordinal1\ X0) \wedge (v7\_ordinal1\ X1)) \Rightarrow (k6\_nat\_d\ X0\ X1 = k6\_nat\_d\ X1\ X0) \quad (18)$$

Assume the following.

$$\forall X0. \forall X1. ((v7\_ordinal1\ X0) \wedge (v7\_ordinal1\ X1)) \Rightarrow (k5\_nat\_d\ X0\ X1 = k5\_nat\_d\ X1\ X0) \quad (19)$$

Assume the following.

$$\forall X0. (v6\_membered\ X0) \Rightarrow (\forall X1. (m1\_subset\_1\ X1\ X0) \Rightarrow (v7\_ordinal1\ X1)) \quad (20)$$

Assume the following.

$$\forall X0. (l3\_lattices\ X0) \Rightarrow ((v3\_lattices\ X0) \Rightarrow (X0 = g3\_lattices\ (u1\_struct\_0\ X0)\ (u2\_lattices\ X0)\ (u1\_lattices\ X0))) \quad (21)$$

**Theorem 1**

$$\forall X0. (m1\_subset\_1\ X0\ (u1\_struct\_0\ k3\_nat\_lat)) \Rightarrow (\forall X1. (m1\_subset\_1\ X1\ (u1\_struct\_0\ k3\_nat\_lat)) \Rightarrow ((k1\_binop\_1\ k1\_nat\_lat\ X0\ (k1\_binop\_1\ k2\_nat\_lat\ X0\ X1) = X0) \wedge ((k1\_binop\_1\ k1\_nat\_lat\ (k1\_binop\_1\ k2\_nat\_lat\ X1\ X0)\ X0 = X0) \wedge ((k1\_binop\_1\ k1\_nat\_lat\ X0\ (k1\_binop\_1\ k2\_nat\_lat\ X1\ X0) = X0) \wedge (k1\_binop\_1\ k1\_nat\_lat\ (k1\_binop\_1\ k2\_nat\_lat\ X0\ X1)\ X0 = X0))))))$$