

## t12\_gcd\_1

(TMXTBniA1xFC7nSXFuYxJ3jfnS3T2EeT5aZ)

October 27, 2020

Let  $v2\_struct\_0 : \iota \Rightarrow o$  be given. Let  $v13\_algstr\_0 : \iota \Rightarrow o$  be given. Let  $v3\_group\_1 : \iota \Rightarrow o$  be given. Let  $v5\_group\_1 : \iota \Rightarrow o$  be given. Let  $v4\_vectsp\_1 : \iota \Rightarrow o$  be given. Let  $v5\_vectsp\_1 : \iota \Rightarrow o$  be given. Let  $v2\_rlvect\_1 : \iota \Rightarrow o$  be given. Let  $v3\_rlvect\_1 : \iota \Rightarrow o$  be given. Let  $v4\_rlvect\_1 : \iota \Rightarrow o$  be given. Let  $v1\_vectsp\_2 : \iota \Rightarrow o$  be given. Let  $l6\_algstr\_0 : \iota \Rightarrow o$  be given. Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $u1\_struct\_0 : \iota \Rightarrow \iota$  be given. Let  $r2\_gcd\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k3\_rlvect\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k4\_struct\_0 : \iota \Rightarrow \iota$  be given. Let  $k1\_gcd\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k5\_struct\_0 : \iota \Rightarrow \iota$  be given. Let  $k8\_group\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v3\_vectsp\_1 : \iota \Rightarrow o$  be given. Let  $l4\_algstr\_0 : \iota \Rightarrow o$  be given. Let  $l3\_algstr\_0 : \iota \Rightarrow o$  be given. Let  $k6\_algstr\_0 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $l1\_algstr\_0 : \iota \Rightarrow o$  be given. Let  $k1\_algstr\_0 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $l2\_algstr\_0 : \iota \Rightarrow o$  be given. Let  $l5\_algstr\_0 : \iota \Rightarrow o$  be given. Let  $l4\_struct\_0 : \iota \Rightarrow o$  be given. Let  $l3\_struct\_0 : \iota \Rightarrow o$  be given. Let  $l2\_struct\_0 : \iota \Rightarrow o$  be given. Let  $v2\_vectsp\_1 : \iota \Rightarrow o$  be given. Let  $v6\_vectsp\_1 : \iota \Rightarrow o$  be given. Let  $v1\_vectsp\_1 : \iota \Rightarrow o$  be given. Assume the following.

$$\begin{aligned}
 & \forall X0. ((\neg v2\_struct\_0 X0) \wedge ((v13\_algstr\_0 X0) \wedge ((v3\_group\_1 \\
 & X0) \wedge ((v5\_group\_1 X0) \wedge ((v4\_vectsp\_1 X0) \wedge ((v5\_vectsp\_1 X0) \wedge ( \\
 & (v2\_rlvect\_1 X0) \wedge ((v3\_rlvect\_1 X0) \wedge ((v4\_rlvect\_1 X0) \wedge ((v1\_vectsp\_2 \\
 & X0) \wedge (l6\_algstr\_0 X0)))))))))) \Rightarrow (\forall X1. (m1\_subset\_1 X1 \\
 & (u1\_struct\_0 X0)) \Rightarrow ((X1 \neq k4\_struct\_0 X0) \Rightarrow (k1\_gcd\_1 X0 X1 X1 = k5\_struct\_0 \\
 & X0)))
 \end{aligned}
 \tag{1}$$

Assume the following.

$$\begin{aligned}
& \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v13\_algstr\_0 X0) \wedge ((v3\_group\_1 \\
& X0) \wedge ((v5\_group\_1 X0) \wedge ((v4\_vectsp\_1 X0) \wedge ((v5\_vectsp\_1 X0) \wedge ( \\
& (v2\_rlvect\_1 X0) \wedge ((v3\_rlvect\_1 X0) \wedge ((v4\_rlvect\_1 X0) \wedge ((v1\_vectsp\_2 \\
& X0) \wedge (l6\_algstr\_0 X0)))))))))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 \\
& (u1\_struct\_0 X0)) \Rightarrow (\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow \\
& (\forall X3.(m1\_subset\_1 X3 (u1\_struct\_0 X0)) \Rightarrow ((X3 \neq k4\_struct\_0 \\
& X0) \Rightarrow (((r2\_gcd\_1 X0 X3 (k8\_group\_1 X0 X1 X2)) \wedge (r2\_gcd\_1 X0 X3 X1)) \Rightarrow \\
& (k1\_gcd\_1 X0 (k8\_group\_1 X0 X1 X2) X3 = k8\_group\_1 X0 (k1\_gcd\_1 X0 \\
& X1 X3) X2)) \wedge (((r2\_gcd\_1 X0 X3 (k8\_group\_1 X0 X1 X2)) \wedge (r2\_gcd\_1 X0 \\
& X3 X2)) \Rightarrow (k1\_gcd\_1 X0 (k8\_group\_1 X0 X1 X2) X3 = k8\_group\_1 X0 X1 (k1\_gcd\_1 \\
& X0 X2 X3)))))))))
\end{aligned} \tag{2}$$

Assume the following.

$$\begin{aligned}
& \forall X0. \forall X1. \forall X2. (((\neg v2\_struct\_0 X0) \wedge ((v3\_vectsp\_1 \\
& X0) \wedge (l4\_algstr\_0 X0))) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge ( \\
& m1\_subset\_1 X2 (u1\_struct\_0 X0))) \Rightarrow (r2\_gcd\_1 X0 X1 X1)
\end{aligned} \tag{3}$$

Assume the following.

$$\begin{aligned}
& \forall X0. \forall X1. \forall X2. (((\neg v2\_struct\_0 X0) \wedge ((v5\_group\_1 \\
& X0) \wedge (l3\_algstr\_0 X0))) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge ( \\
& m1\_subset\_1 X2 (u1\_struct\_0 X0))) \Rightarrow (k8\_group\_1 X0 X1 X2 = k6\_algstr\_0 \\
& X0 X1 X2)
\end{aligned} \tag{4}$$

Assume the following.

$$\begin{aligned}
& \forall X0. \forall X1. \forall X2. (((v2\_rlvect\_1 X0) \wedge (l1\_algstr\_0 \\
& X0)) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge (m1\_subset\_1 X2 (u1\_struct\_0 \\
& X0)))) \Rightarrow (k3\_rlvect\_1 X0 X1 X2 = k1\_algstr\_0 X0 X1 X2)
\end{aligned} \tag{5}$$

Assume the following.

$$\forall X0. (l6\_algstr\_0 X0) \Rightarrow ((l2\_algstr\_0 X0) \wedge (l5\_algstr\_0 X0)) \tag{6}$$

Assume the following.

$$\forall X0. (l5\_algstr\_0 X0) \Rightarrow ((l4\_algstr\_0 X0) \wedge (l4\_struct\_0 X0)) \tag{7}$$

Assume the following.

$$\forall X0. (l4\_algstr\_0 X0) \Rightarrow ((l3\_struct\_0 X0) \wedge (l3\_algstr\_0 X0)) \tag{8}$$

Assume the following.

$$\forall X0. (l2\_algstr\_0 X0) \Rightarrow ((l2\_struct\_0 X0) \wedge (l1\_algstr\_0 X0)) \tag{9}$$

Assume the following.

$$\forall X0.(l3\_struct\_0 X0) \Rightarrow (m1\_subset\_1 (k5\_struct\_0 X0) (u1\_struct\_0 X0)) \quad (10)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0) \wedge ((v13\_algstr\_0 \\ & X0) \wedge ((v3\_group\_1 X0) \wedge ((v5\_group\_1 X0) \wedge ((v4\_vectsp\_1 X0) \wedge (( \\ & v5\_vectsp\_1 X0) \wedge ((v2\_rlvect\_1 X0) \wedge ((v3\_rlvect\_1 X0) \wedge ((v4\_rlvect\_1 \\ & X0) \wedge ((v1\_vectsp\_2 X0) \wedge (l6\_algstr\_0 X0)))))))))) \wedge ((m1\_subset\_1 \\ & X1 (u1\_struct\_0 X0) \wedge (m1\_subset\_1 X2 (u1\_struct\_0 X0)))) \Rightarrow (m1\_subset\_1 \\ & (k1\_gcd\_1 X0 X1 X2) (u1\_struct\_0 X0)) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.((l1\_algstr\_0 X0) \wedge ((m1\_subset\_1 \\ & X1 (u1\_struct\_0 X0) \wedge (m1\_subset\_1 X2 (u1\_struct\_0 X0)))) \Rightarrow (m1\_subset\_1 \\ & (k1\_algstr\_0 X0 X1 X2) (u1\_struct\_0 X0)) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} & \forall X0.((\neg v2\_struct\_0 X0) \wedge (l4\_algstr\_0 X0)) \Rightarrow ((v3\_vectsp\_1 \\ & X0) \Leftrightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 X0)) \Rightarrow (k6\_algstr\_0 \\ & X0 X1 (k5\_struct\_0 X0) = X1))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} & \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v13\_algstr\_0 X0) \wedge ((v3\_group\_1 \\ & X0) \wedge ((v5\_group\_1 X0) \wedge ((v4\_vectsp\_1 X0) \wedge ((v5\_vectsp\_1 X0) \wedge ( \\ & (v2\_rlvect\_1 X0) \wedge ((v3\_rlvect\_1 X0) \wedge ((v4\_rlvect\_1 X0) \wedge ((v1\_vectsp\_2 \\ & X0) \wedge (l6\_algstr\_0 X0)))))))))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 \\ & (u1\_struct\_0 X0)) \Rightarrow (\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow \\ & ((r2\_gcd\_1 X0 X2 X1) \Rightarrow ((X2 = k4\_struct\_0 X0) \vee (\forall X3.(m1\_subset\_1 \\ & X3 (u1\_struct\_0 X0)) \Rightarrow ((X3 = k1\_gcd\_1 X0 X1 X2) \Leftrightarrow (k8\_group\_1 X0 X3 \\ & X2 = X1)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} & \forall X0.((\neg v2\_struct\_0 X0) \wedge (l6\_algstr\_0 X0)) \Rightarrow ((v2\_vectsp\_1 \\ & X0) \Leftrightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 X0)) \Rightarrow (\forall X2. \\ & (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (\forall X3.(m1\_subset\_1 X3 \\ & (u1\_struct\_0 X0)) \Rightarrow (k6\_algstr\_0 X0 (k1\_algstr\_0 X0 X2 X3) X1 = k1\_algstr\_0 \\ & X0 (k6\_algstr\_0 X0 X2 X1) (k6\_algstr\_0 X0 X3 X1)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge(v5\_group\_1 X0)\wedge(l3\_algstr\_0 X0))\wedge((m1\_subset\_1 X1 (u1\_struct\_0 X0))\wedge(m1\_subset\_1 X2 (u1\_struct\_0 X0))))\Rightarrow(k8\_group\_1 X0 X1 X2 = k8\_group\_1 X0 X2 X1) \quad (16)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((v2\_rlvect\_1 X0)\wedge(l1\_algstr\_0 X0))\wedge((m1\_subset\_1 X1 (u1\_struct\_0 X0))\wedge(m1\_subset\_1 X2 (u1\_struct\_0 X0))))\Rightarrow(k3\_rlvect\_1 X0 X1 X2 = k3\_rlvect\_1 X0 X2 X1) \quad (17)$$

Assume the following.

$$\forall X0.(l4\_algstr\_0 X0)\Rightarrow(((\neg v2\_struct\_0 X0)\wedge(v4\_vectsp\_1 X0))\Rightarrow((\neg v2\_struct\_0 X0)\wedge((v3\_vectsp\_1 X0)\wedge(v6\_vectsp\_1 X0)))) \quad (18)$$

Assume the following.

$$\forall X0.(l6\_algstr\_0 X0)\Rightarrow(((\neg v2\_struct\_0 X0)\wedge(v5\_vectsp\_1 X0))\Rightarrow((\neg v2\_struct\_0 X0)\wedge((v1\_vectsp\_1 X0)\wedge(v2\_vectsp\_1 X0)))) \quad (19)$$

**Theorem 1**

$$\begin{aligned} & \forall X0.((\neg v2\_struct\_0 X0)\wedge((v13\_algstr\_0 X0)\wedge((v3\_group\_1 X0)\wedge((v5\_group\_1 X0)\wedge((v4\_vectsp\_1 X0)\wedge((v5\_vectsp\_1 X0)\wedge((v2\_rlvect\_1 X0)\wedge((v3\_rlvect\_1 X0)\wedge((v4\_rlvect\_1 X0)\wedge((v1\_vectsp\_2 X0)\wedge(l6\_algstr\_0 X0))))))))))\Rightarrow(\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 X0))\Rightarrow(\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 X0))\Rightarrow(\forall X3.(m1\_subset\_1 X3 (u1\_struct\_0 X0))\Rightarrow(((r2\_gcd\_1 X0 X3 X1)\wedge((r2\_gcd\_1 X0 X3 X2)\wedge(r2\_gcd\_1 X0 X3 (k3\_rlvect\_1 X0 X1 X2))))\Rightarrow((X3 = k4\_struct\_0 X0)\vee(k3\_rlvect\_1 X0 (k1\_gcd\_1 X0 X1 X3) (k1\_gcd\_1 X0 X2 X3) = k1\_gcd\_1 X0 (k3\_rlvect\_1 X0 X1 X2) X3)))))) \end{aligned}$$