

## t15\_closure1

(TMYVfvKpn71KtLjwxLNv6ATZJ4K42xwBJf7)

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Let  $v1\_relat\_1 : \iota \Rightarrow o$  be given. Let  $v4\_relat\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v1\_funct\_1 : \iota \Rightarrow o$  be given. Let  $v1\_partfun1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v4\_closure1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k2\_msualg\_3 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k5\_mssubfam : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $m2\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $r6\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k1\_mboolean : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $r1\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $r2\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k2\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v2\_relat\_1 : \iota \Rightarrow o$  be given. Let  $m1\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k16\_pralg\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_closure1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k2\_closure1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k15\_pralg\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v1\_mssubfam : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v2\_mssubfam : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v3\_mssubfam : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v4\_mssubfam : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v5\_mssubfam : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v6\_mssubfam : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $m3\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Assume the following.

$$\begin{aligned}
 & \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\
 & \quad (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1\_relat\_1 \\
 & X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\
 & \quad ((r6\_pboole X0 X1 (k1\_mboolean X0 X2)) \Leftrightarrow (\forall X3. ((v1\_relat\_1 \\
 & X3) \wedge ((v4\_relat\_1 X3 X0) \wedge ((v1\_funct\_1 X3) \wedge (v1\_partfun1 X3 X0)))) \Rightarrow \\
 & \quad ((r1\_pboole X0 X3 X1) \Leftrightarrow (r2\_pboole X0 X3 X2))))
 \end{aligned} \tag{1}$$

Assume the following.

$$\begin{aligned}
 & \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\
 & \quad (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1\_relat\_1 \\
 & X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\
 & \quad (\forall X3. ((v1\_relat\_1 X3) \wedge ((v4\_relat\_1 X3 X0) \wedge ((v1\_funct\_1 \\
 & X3) \wedge (v1\_partfun1 X3 X0)))) \Rightarrow (((r2\_pboole X0 X1 X2) \wedge (r2\_pboole \\
 & X0 X3 X2)) \Rightarrow (r2\_pboole X0 (k2\_pboole X0 X1 X3) X2))))
 \end{aligned} \tag{2}$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\ & (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1\_relat\_1 \\ & X2) \wedge ((v2\_relat\_1 X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge \\ & v1\_partfun1 X2 X0)))) \Rightarrow ((m1\_pboole X1 X0 X2) \Rightarrow (r1\_pboole X0 X1 X2))) \end{aligned} \quad (3)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\ & (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1\_relat\_1 \\ & X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\ & (r1\_pboole X0 X1 X2) \Rightarrow (m1\_pboole X1 X0 X2)) \end{aligned} \quad (4)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v2\_relat\_1 X1) \wedge ((v4\_relat\_1 \\ & X1 X0) \wedge ((v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0)))) \Rightarrow (\forall X2. \\ & (m1\_pboole X2 X0 X1) \Rightarrow (\forall X3. (m1\_pboole X3 X0 X1) \Rightarrow ((m1\_pboole \\ & (k2\_pboole X0 X2 X3) X0 X1) \Rightarrow (r6\_pboole X0 (k16\_pralg\_1 X0 (k2\_msualg\_3 \\ & X0 X1) (k2\_pboole X0 X2 X3)) (k2\_pboole X0 (k1\_closure1 X0 X1 X1 (k2\_msualg\_3 \\ & X0 X1) X2) (k1\_closure1 X0 X1 X1 (k2\_msualg\_3 X0 X1) X3)))))) \end{aligned} \quad (5)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 \\ & X1 X0) \wedge ((v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0)))) \wedge ((v1\_relat\_1 \\ & X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\ & (r6\_pboole X0 X1 X2) \Leftrightarrow (X1 = X2) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\ & (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (k5\_mssubfam X0 X1 = k1\_mboolean \\ & X0 X1) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. \forall X3. (((v1\_relat\_1 X1) \wedge \\ & ((v4\_relat\_1 X1 X0) \wedge ((v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0)))) \wedge \\ & ((m2\_pboole X2 X0 (k5\_mssubfam X0 X1) (k5\_mssubfam X0 X1)) \wedge (m1\_pboole \\ & X3 X0 (k5\_mssubfam X0 X1)))) \Rightarrow (k2\_closure1 X0 X1 X2 X3 = k15\_pralg\_1 \\ & X2 X3) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.\forall X3.\forall X4.(((v1\_relat\_1 \\ & X1)\wedge((v2\_relat\_1 X1)\wedge((v4\_relat\_1 X1 X0)\wedge((v1\_funct\_1 X1)\wedge \\ & v1\_partfun1 X1 X0))))\wedge(((v1\_relat\_1 X2)\wedge((v2\_relat\_1 X2)\wedge \\ & (v4\_relat\_1 X2 X0)\wedge((v1\_funct\_1 X2)\wedge(v1\_partfun1 X2 X0))))\wedge \\ & ((m2\_pboole X3 X0 X1 X2)\wedge(m1\_pboole X4 X0 X1)))\Rightarrow(k1\_closure1 X0 \\ & X1 X2 X3 X4 = k15\_pralg\_1 X3 X4) \end{aligned} \tag{9}$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.((v1\_relat\_1 X1)\wedge((v4\_relat\_1 X1 X0)\wedge \\ & (v1\_funct\_1 X1)\wedge(v1\_partfun1 X1 X0)))\Rightarrow(\forall X2.(m1\_pboole \\ & X2 X0 X1)\Rightarrow((v1\_relat\_1 X2)\wedge((v4\_relat\_1 X2 X0)\wedge((v1\_funct\_1 X2)\wedge \\ & (v1\_partfun1 X2 X0)))))) \end{aligned} \tag{10}$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.((v1\_relat\_1 X1)\wedge((v4\_relat\_1 X1 X0)\wedge \\ & (v1\_funct\_1 X1)\wedge(v1\_partfun1 X1 X0)))\Rightarrow((v1\_mssubfam (k5\_mssubfam \\ & X0 X1) X0 X1)\wedge((v2\_mssubfam (k5\_mssubfam X0 X1) X0 X1)\wedge((v3\_mssubfam \\ & (k5\_mssubfam X0 X1) X0 X1)\wedge((v4\_mssubfam (k5\_mssubfam X0 X1) X0 \\ & X1)\wedge((v5\_mssubfam (k5\_mssubfam X0 X1) X0 X1)\wedge((v6\_mssubfam (k5\_mssubfam \\ & X0 X1) X0 X1)\wedge(m3\_pboole (k5\_mssubfam X0 X1) X0 (k1\_mboolean X0 X1)))))))))) \end{aligned} \tag{11}$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(((v1\_relat\_1 X1)\wedge((v4\_relat\_1 \\ & X1 X0)\wedge((v1\_funct\_1 X1)\wedge(v1\_partfun1 X1 X0))))\wedge(((v1\_relat\_1 \\ & X2)\wedge((v4\_relat\_1 X2 X0)\wedge((v1\_funct\_1 X2)\wedge(v1\_partfun1 X2 X0))))))\Rightarrow \\ & ((v1\_relat\_1 (k2\_pboole X0 X1 X2))\wedge((v4\_relat\_1 (k2\_pboole X0 \\ & X1 X2) X0)\wedge((v1\_funct\_1 (k2\_pboole X0 X1 X2))\wedge(v1\_partfun1 (k2\_pboole \\ & X0 X1 X2) X0)))))) \end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.((v1\_relat\_1 X1)\wedge((v4\_relat\_1 X1 X0)\wedge \\ & (v1\_funct\_1 X1)\wedge(v1\_partfun1 X1 X0)))\Rightarrow(m2\_pboole (k2\_msualg\_3 \\ & X0 X1) X0 X1 X1) \end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.((v1\_relat\_1 X1)\wedge((v4\_relat\_1 X1 X0)\wedge \\ & (v1\_funct\_1 X1)\wedge(v1\_partfun1 X1 X0)))\Rightarrow((v1\_relat\_1 (k1\_mboolean \\ & X0 X1))\wedge((v4\_relat\_1 (k1\_mboolean X0 X1) X0)\wedge((v1\_funct\_1 (k1\_mboolean \\ & X0 X1))\wedge(v1\_partfun1 (k1\_mboolean X0 X1) X0)))))) \end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned}
& \forall X0.\forall X1.((v1\_relat\_1 X1)\wedge((v4\_relat\_1 X1 X0)\wedge \\
& (v1\_funct\_1 X1)\wedge(v1\_partfun1 X1 X0))))\Rightarrow(\forall X2.(m2\_pboole \\
& X2 X0 (k5\_mssubfam X0 X1) (k5\_mssubfam X0 X1))\Rightarrow((v4\_closure1 X2 \\
& X0 X1)\Leftrightarrow(\forall X3.(m1\_pboole X3 X0 (k5\_mssubfam X0 X1))\Rightarrow(\forall X4. \\
& (m1\_pboole X4 X0 (k5\_mssubfam X0 X1))\Rightarrow(r6\_pboole X0 (k16\_pralg\_1 \\
& X0 X2 (k2\_pboole X0 X3 X4) (k2\_pboole X0 (k2\_closure1 X0 X1 X2 X3) \\
& (k2\_closure1 X0 X1 X2 X4)))))))))
\end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned}
& \forall X0.\forall X1.((v1\_relat\_1 X1)\wedge((v4\_relat\_1 X1 X0)\wedge \\
& (v1\_funct\_1 X1)\wedge(v1\_partfun1 X1 X0))))\Rightarrow(\forall X2.(m3\_pboole \\
& X2 X0 (k1\_mboolean X0 X1))\Rightarrow((v6\_mssubfam X2 X0 X1)\Rightarrow(v2\_relat\_1 \\
& X2)))
\end{aligned} \tag{16}$$

**Theorem 1**

$$\begin{aligned}
& \forall X0.\forall X1.((v1\_relat\_1 X1)\wedge((v4\_relat\_1 X1 X0)\wedge \\
& (v1\_funct\_1 X1)\wedge(v1\_partfun1 X1 X0))))\Rightarrow((v4\_closure1 (k2\_msualg\_3 \\
& X0 (k5\_mssubfam X0 X1) X0 X1)\wedge(m2\_pboole (k2\_msualg\_3 X0 (k5\_mssubfam \\
& X0 X1) X0 (k5\_mssubfam X0 X1) (k5\_mssubfam X0 X1)))
\end{aligned}$$