

# t15\_msafree3 (TM- REd8QxvE2ud6Qr9mFSWAipjWZN2p4of6H)

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Let  $v11\_struct\_0 : \iota \Rightarrow o$  be given. Let  $v1\_instalg1 : \iota \Rightarrow o$  be given. Let  $l1\_msualg\_1 : \iota \Rightarrow o$  be given. Let  $v1\_relat\_1 : \iota \Rightarrow o$  be given. Let  $v2\_relat\_1 : \iota \Rightarrow o$  be given. Let  $v4\_relat\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $u1\_struct\_0 : \iota \Rightarrow \iota$  be given. Let  $v1\_funct\_1 : \iota \Rightarrow o$  be given. Let  $v1\_partfun1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $m1\_dtconstr : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k5\_msafree : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k5\_trees\_3 : \iota \Rightarrow \iota$  be given. Let  $k1\_msaterm : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $r8\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k4\_msafree3 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k3\_msafree3 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $r2\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $r6\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k3\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k2\_msafree3 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v1\_xboole\_0 : \iota \Rightarrow o$  be given. Let  $v2\_struct\_0 : \iota \Rightarrow o$  be given. Let  $v1\_lang1 : \iota \Rightarrow o$  be given. Let  $l1\_struct\_0 : \iota \Rightarrow o$  be given. Let  $m1\_trees\_3 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k1\_zfmisc\_1 : \iota \Rightarrow \iota$  be given. Let  $l5\_struct\_0 : \iota \Rightarrow o$  be given. Let  $l1\_lang1 : \iota \Rightarrow o$  be given. Let  $v3\_trees\_3 : \iota \Rightarrow o$  be given. Let  $v3\_trees\_2 : \iota \Rightarrow o$  be given. Let  $v4\_funct\_1 : \iota \Rightarrow o$  be given. Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\ & (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1\_relat\_1 \\ & X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\ & ((r2\_pboole X0 X1 X2) \Rightarrow (r6\_pboole X0 (k3\_pboole X0 X1 X2) X1))) \end{aligned} \quad (1)$$

Assume the following.

$$\begin{aligned} & \forall X0. ((\neg v11\_struct\_0 X0) \wedge ((v1\_instalg1 X0) \wedge (l1\_msualg\_1 \\ & X0))) \Rightarrow (\forall X1. ((v1\_relat\_1 X1) \wedge ((v2\_relat\_1 X1) \wedge ((v4\_relat\_1 \\ & X1 (u1\_struct\_0 X0)) \wedge ((v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 (u1\_struct\_0 \\ & X0)))))) \Rightarrow (\forall X2. (m1\_dtconstr X2 (u1\_struct\_0 (k5\_msafree \\ & X0 X1)) (k5\_trees\_3 (u1\_struct\_0 (k5\_msafree X0 X1))) (k1\_msaterm \\ & X0 X1))) \Rightarrow (r2\_pboole (u1\_struct\_0 X0) (k2\_msafree3 X0 X2) X1))) \end{aligned} \quad (2)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.((\neg v1\_xboole\_0 X0)\wedge(((v1\_relat\_1 \\ & X1)\wedge((v4\_relat\_1 X1 X0)\wedge((v1\_funct\_1 X1)\wedge(v1\_partfun1 X1 X0))))\wedge \\ & ((v1\_relat\_1 X2)\wedge((v4\_relat\_1 X2 X0)\wedge((v1\_funct\_1 X2)\wedge(v1\_partfun1 \\ & X2 X0))))))\Rightarrow((r8\_pboole X0 X1 X2)\Leftrightarrow(X1 = X2)) \end{aligned} \quad (3)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(((v1\_relat\_1 X1)\wedge((v4\_relat\_1 \\ & X1 X0)\wedge((v1\_funct\_1 X1)\wedge(v1\_partfun1 X1 X0))))\wedge((v1\_relat\_1 \\ & X2)\wedge((v4\_relat\_1 X2 X0)\wedge((v1\_funct\_1 X2)\wedge(v1\_partfun1 X2 X0))))\Rightarrow \\ & ((r6\_pboole X0 X1 X2)\Leftrightarrow(X1 = X2)) \end{aligned} \quad (4)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.(((\neg v2\_struct\_0 X0)\wedge((\neg v11\_struct\_0 X0)\wedge \\ & (l1\_msualg\_1 X0)))\wedge((v1\_relat\_1 X1)\wedge((v4\_relat\_1 X1 (u1\_struct\_0 \\ & X0))\wedge((v1\_funct\_1 X1)\wedge(v1\_partfun1 X1 (u1\_struct\_0 X0))))))\Rightarrow \\ & ((\neg v2\_struct\_0 (k5\_msafree X0 X1))\wedge(v1\_lang1 (k5\_msafree X0 X1))) \end{aligned} \quad (5)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0)\wedge(l1\_struct\_0 X0))\Rightarrow(\neg v1\_xboole\_0 (u1\_struct\_0 X0)) \quad (6)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.(((\neg v2\_struct\_0 X0)\wedge((\neg v11\_struct\_0 X0)\wedge \\ & (l1\_msualg\_1 X0)))\wedge((v1\_relat\_1 X1)\wedge((v2\_relat\_1 X1)\wedge((v4\_relat\_1 \\ & X1 (u1\_struct\_0 X0))\wedge((v1\_funct\_1 X1)\wedge(v1\_partfun1 X1 (u1\_struct\_0 \\ & X0))))))\Rightarrow(\neg v1\_xboole\_0 (k1\_msaterm X0 X1)) \end{aligned} \quad (7)$$

Assume the following.

$$\forall X0.(\neg v1\_xboole\_0 X0)\Rightarrow(\forall X1.(m1\_trees\_3 X1 X0)\Rightarrow(\neg v1\_xboole\_0 X1)) \quad (8)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.((\neg v1\_xboole\_0 X0)\wedge(((\neg v1\_xboole\_0 \\ & X1)\wedge(m1\_trees\_3 X1 X0))\wedge((\neg v1\_xboole\_0 X2)\wedge(m1\_subset\_1 X2 ( \\ & k1\_zfmisc\_1 X1))))))\Rightarrow(\forall X3.(m1\_dtconstr X3 X0 X1 X2)\Rightarrow(m1\_subset\_1 \\ & X3 X1)) \end{aligned} \quad (9)$$

Assume the following.

$$\forall X0.(l5\_struct\_0 X0)\Rightarrow(l1\_struct\_0 X0) \quad (10)$$

Assume the following.

$$\forall X0.(l1\_msualg\_1 X0) \Rightarrow (l5\_struct\_0 X0) \quad (11)$$

Assume the following.

$$\forall X0.(l1\_lang1 X0) \Rightarrow (l1\_struct\_0 X0) \quad (12)$$

Assume the following.

$$\forall X0.(\neg v1\_xboole\_0 X0) \Rightarrow (m1\_trees\_3 (k5\_trees\_3 X0) X0) \quad (13)$$

Assume the following.

$$\begin{aligned} \forall X0.\forall X1.(((\neg v2\_struct\_0 X0) \wedge ((\neg v11\_struct\_0 X0) \wedge \\ (l1\_msualg\_1 X0))) \wedge ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 (u1\_struct\_0 \\ X0)) \wedge ((v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 (u1\_struct\_0 X0)))))) \Rightarrow \\ (l1\_lang1 (k5\_msafree X0 X1)) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} \forall X0.\forall X1.\forall X2.(((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 \\ X1 X0) \wedge ((v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0)))) \wedge ((v1\_relat\_1 \\ X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\ ((v1\_relat\_1 (k3\_pboole X0 X1 X2)) \wedge ((v4\_relat\_1 (k3\_pboole X0 \\ X1 X2) X0) \wedge ((v1\_funct\_1 (k3\_pboole X0 X1 X2)) \wedge (v1\_partfun1 (k3\_pboole \\ X0 X1 X2) X0)))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} \forall X0.\forall X1.((l1\_msualg\_1 X0) \wedge ((\neg v1\_xboole\_0 X1) \wedge \\ (v1\_relat\_1 X1))) \Rightarrow ((v1\_relat\_1 (k2\_msafree3 X0 X1)) \wedge ((v4\_relat\_1 \\ (k2\_msafree3 X0 X1) (u1\_struct\_0 X0)) \wedge ((v1\_funct\_1 (k2\_msafree3 \\ X0 X1)) \wedge (v1\_partfun1 (k2\_msafree3 X0 X1) (u1\_struct\_0 X0)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} \forall X0.\forall X1.(((\neg v2\_struct\_0 X0) \wedge ((\neg v11\_struct\_0 X0) \wedge \\ (l1\_msualg\_1 X0))) \wedge ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 (u1\_struct\_0 \\ X0)) \wedge ((v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 (u1\_struct\_0 X0)))))) \Rightarrow \\ (m1\_subset\_1 (k1\_msaterm X0 X1) (k1\_zfmisc\_1 (k5\_trees\_3 (u1\_struct\_0 \\ (k5\_msafree X0 X1)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} \forall X0.(((\neg v11\_struct\_0 X0) \wedge ((v1\_instalg1 X0) \wedge (l1\_msualg\_1 \\ X0))) \Rightarrow (\forall X1.(((v1\_relat\_1 X1) \wedge ((v2\_relat\_1 X1) \wedge ((v4\_relat\_1 \\ X1 (u1\_struct\_0 X0)) \wedge ((v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 (u1\_struct\_0 \\ X0)))))) \Rightarrow (\forall X2.(m1\_dtconstr X2 (u1\_struct\_0 (k5\_msafree \\ X0 X1)) (k5\_trees\_3 (u1\_struct\_0 (k5\_msafree X0 X1))) (k1\_msaterm \\ X0 X1)) \Rightarrow (k4\_msafree3 X0 X1 X2 = k2\_msafree3 X0 X2)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} \forall X0.(l1\_msualg\_1 X0) \Rightarrow (\forall X1.((v1\_relat\_1 X1) \wedge (( \\ v4\_relat\_1 X1 (u1\_struct\_0 X0) \wedge ((v1\_funct\_1 X1) \wedge (v1\_partfun1 \\ X1 (u1\_struct\_0 X0)))))) \Rightarrow (\forall X2.((\neg v1\_xboole\_0 X2) \wedge (v1\_relat\_1 \\ X2)) \Rightarrow (k3\_msafree3 X0 X1 X2 = k3\_pboole (u1\_struct\_0 X0) X1 (k2\_msafree3 \\ X0 X2)))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} \forall X0.\forall X1.\forall X2.(((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 \\ X1 X0) \wedge ((v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0)))) \wedge ((v1\_relat\_1 \\ X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))))) \Rightarrow \\ (k3\_pboole X0 X1 X2 = k3\_pboole X0 X2 X1) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} \forall X0.((\neg v1\_xboole\_0 X0) \wedge (v3\_trees\_3 X0)) \Rightarrow (\forall X1. \\ (m1\_subset\_1 X1 X0) \Rightarrow (v3\_trees\_2 X1)) \end{aligned} \quad (21)$$

Assume the following.

$$\forall X0.(v3\_trees\_3 X0) \Rightarrow (v4\_funct\_1 X0) \quad (22)$$

Assume the following.

$$\begin{aligned} \forall X0.(v4\_funct\_1 X0) \Rightarrow (\forall X1.(m1\_subset\_1 X1 X0) \Rightarrow ( \\ (v1\_relat\_1 X1) \wedge (v1\_funct\_1 X1))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} \forall X0.((v1\_relat\_1 X0) \wedge ((v1\_funct\_1 X0) \wedge (v3\_trees\_2 X0))) \Rightarrow \\ ((\neg v1\_xboole\_0 X0) \wedge ((v1\_relat\_1 X0) \wedge ((v1\_funct\_1 X0) \wedge (v3\_trees\_2 \\ X0)))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} \forall X0.(l1\_msualg\_1 X0) \Rightarrow (((v2\_struct\_0 X0) \wedge (v1\_instalg1 \\ X0)) \Rightarrow (v11\_struct\_0 X0)) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} \forall X0.(v1\_xboole\_0 X0) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (k1\_zfmisc.1 \\ X0)) \Rightarrow (v1\_xboole\_0 X1)) \end{aligned} \quad (26)$$

Assume the following.

$$\forall X0.(v1\_xboole\_0 X0) \Rightarrow (v1\_relat\_1 X0) \quad (27)$$

Assume the following.

$$\forall X0.(v1\_xboole\_0 X0) \Rightarrow (v1\_funct\_1 X0) \quad (28)$$

Assume the following.

$$\forall X0.(\neg v1\_xboole\_0 X0) \Rightarrow (\forall X1.(m1\_trees\_3 X1 X0) \Rightarrow (v3\_trees\_3 X1)) \quad (29)$$

**Theorem 1**

$$\begin{aligned} & \forall X0.((\neg v11\_struct\_0 X0) \wedge ((v1\_instalg1 X0) \wedge (l1\_msualg1 \\ & X0))) \Rightarrow (\forall X1.((v1\_relat\_1 X1) \wedge ((v2\_relat\_1 X1) \wedge ((v4\_relat\_1 \\ & X1 (u1\_struct\_0 X0)) \wedge ((v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 (u1\_struct\_0 \\ & X0)))))) \Rightarrow (\forall X2.(m1\_dtconstr X2 (u1\_struct\_0 (k5\_msafree \\ & X0 X1)) (k5\_trees\_3 (u1\_struct\_0 (k5\_msafree X0 X1))) (k1\_msaterm \\ & X0 X1)) \Rightarrow (r8\_pboole (u1\_struct\_0 X0) (k4\_msafree3 X0 X1 X2) (k3\_msafree3 \\ & X0 X1 X2)))) \end{aligned}$$