

t15\_msualg\_7  
(TMWhNiteWMDjfm67s6rKX7y8R4papoxvkfy)

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Let  $v2\_struct\_0 : \iota \Rightarrow o$  be given. Let  $v10\_lattices : \iota \Rightarrow o$  be given. Let  $v4\_lattice3 : \iota \Rightarrow o$  be given. Let  $l3\_lattices : \iota \Rightarrow o$  be given. Let  $m2\_nat\_lat : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v2\_msualg\_7 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v1\_xboole\_0 : \iota \Rightarrow o$  be given. Let  $v8\_lattices : \iota \Rightarrow o$  be given. Let  $v9\_lattices : \iota \Rightarrow o$  be given. Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $u1\_struct\_0 : \iota \Rightarrow \iota$  be given. Let  $r1\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k2\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_zfmisc\_1 : \iota \Rightarrow \iota$  be given. Let  $r4\_lattice3 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k3\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k4\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v6\_lattices : \iota \Rightarrow o$  be given. Let  $l1\_lattices : \iota \Rightarrow o$  be given. Let  $l2\_lattices : \iota \Rightarrow o$  be given. Let  $k15\_lattice3 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $r1\_tarski : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $u2\_lattices : \iota \Rightarrow \iota$  be given. Let  $k1\_realset1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $u1\_lattices : \iota \Rightarrow \iota$  be given. Let  $v4\_lattices : \iota \Rightarrow o$  be given. Let  $v5\_lattices : \iota \Rightarrow o$  be given. Let  $v7\_lattices : \iota \Rightarrow o$  be given. Assume the following.

$$\forall X0. \forall X1. \neg (X0 \in X1) \wedge (v1\_xboole\_0 X1) \quad (1)$$

Assume the following.

$$\begin{aligned} \forall X0. ((\neg v2\_struct\_0 X0) \wedge ((v8\_lattices X0) \wedge ((v9\_lattices \\ X0) \wedge (l3\_lattices X0)))) \Rightarrow (\forall X1. (m1\_subset\_1 X1 (u1\_struct\_0 \\ X0)) \Rightarrow (\forall X2. (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow ((r1\_lattices \\ X0 X1 X2) \Leftrightarrow (k2\_lattices X0 X1 X2 = X1)))) \end{aligned} \quad (2)$$

Assume the following.

$$\forall X0. \forall X1. (m1\_subset\_1 X0 X1) \Rightarrow ((v1\_xboole\_0 X1) \vee (X0 \in X1)) \quad (3)$$

Assume the following.

$$\forall X0. \forall X1. (X0 \in X1) \Rightarrow (m1\_subset\_1 X0 X1) \quad (4)$$

Assume the following.

$$\begin{aligned} \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices \\ X0))) \Rightarrow (\forall X1.(m2\_nat\_lat X1 X0) \Rightarrow (\forall X2.(m1\_subset\_1 \\ X2 (k1\_zfmisc.1 (u1\_struct\_0 X1))) \Rightarrow (\forall X3.(m1\_subset\_1 \\ X3 (u1\_struct\_0 X0)) \Rightarrow (\forall X4.(m1\_subset\_1 X4 (u1\_struct\_0 \\ X1)) \Rightarrow ((X3 = X4) \Rightarrow ((r4\_lattice3 X0 X3 X2) \Leftrightarrow (r4\_lattice3 X1 X4 X2))))))) \end{aligned} \quad (5)$$

Assume the following.

$$\begin{aligned} \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices \\ X0))) \Rightarrow (\forall X1.(m2\_nat\_lat X1 X0) \Rightarrow (\forall X2.(m1\_subset\_1 \\ X2 (u1\_struct\_0 X0)) \Rightarrow (\forall X3.(m1\_subset\_1 X3 (u1\_struct\_0 \\ X0)) \Rightarrow (\forall X4.(m1\_subset\_1 X4 (u1\_struct\_0 X1)) \Rightarrow (\forall X5. \\ (m1\_subset\_1 X5 (u1\_struct\_0 X1)) \Rightarrow (((X2 = X4) \wedge (X3 = X5)) \Rightarrow ((k3\_lattices \\ X0 X2 X3 = k3\_lattices X1 X4 X5) \wedge (k4\_lattices X0 X2 X3 = k4\_lattices \\ X1 X4 X5)))))))) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} \forall X0. \forall X1. \forall X2. (((\neg v2\_struct\_0 X0) \wedge ((v6\_lattices \\ X0) \wedge (l1\_lattices X0))) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge ( \\ m1\_subset\_1 X2 (u1\_struct\_0 X0)))) \Rightarrow (k4\_lattices X0 X1 X2 = k2\_lattices \\ X0 X1 X2) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices \\ X0))) \Rightarrow (\forall X1.(m2\_nat\_lat X1 X0) \Rightarrow ((\neg v2\_struct\_0 X1) \wedge ((v10\_lattices \\ X1) \wedge (l3\_lattices X1)))) \end{aligned} \quad (8)$$

Assume the following.

$$\forall X0.(l3\_lattices X0) \Rightarrow ((l1\_lattices X0) \wedge (l2\_lattices X0)) \quad (9)$$

Assume the following.

$$\begin{aligned} \forall X0. \forall X1. ((\neg v2\_struct\_0 X0) \wedge (l3\_lattices X0)) \Rightarrow \\ (m1\_subset\_1 (k15\_lattice3 X0 X1) (u1\_struct\_0 X0)) \end{aligned} \quad (10)$$

Assume the following.

$$\forall X0. \forall X1. (r1\_tarski X0 X1) \Leftrightarrow (\forall X2. (X2 \in X0) \Rightarrow (X2 \in X1)) \quad (11)$$

Assume the following.

$$\begin{aligned} \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices \\ X0))) \Rightarrow (\forall X1.(m2\_nat\_lat X1 X0) \Rightarrow ((v2\_msualg.7 X1 X0) \Leftrightarrow (\forall X2. \\ (m1\_subset\_1 X2 (k1\_zfmisc.1 (u1\_struct\_0 X1))) \Rightarrow (k15\_lattice3 \\ X0 X2 \in u1\_struct\_0 X1)))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned}
& \forall X0.((\neg v2\_struct\_0 X0) \wedge (l3\_lattices X0)) \Rightarrow (((\neg v2\_struct\_0 \\
& X0) \wedge ((v10\_lattices X0) \wedge ((v4\_lattice3 X0) \wedge (l3\_lattices X0)))) \Rightarrow \\
& (\forall X1. \forall X2. (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow ((X2 = \\
& k15\_lattice3 X0 X1) \Leftrightarrow ((r4\_lattice3 X0 X2 X1) \wedge (\forall X3. (m1\_subset\_1 \\
& X3 (u1\_struct\_0 X0)) \Rightarrow ((r4\_lattice3 X0 X3 X1) \Rightarrow (r1\_lattices X0 X2 \\
& X3))))))
\end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned}
& \forall X0.((\neg v2\_struct\_0 X0) \wedge (l3\_lattices X0)) \Rightarrow ((v4\_lattice3 \\
& X0) \Leftrightarrow (\forall X1. (m1\_subset\_1 X1 (k1\_zfmisc\_1 (u1\_struct\_0 X0))) \Rightarrow \\
& (\exists X2. (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \wedge ((r4\_lattice3 \\
& X0 X2 X1) \wedge (\forall X3. (m1\_subset\_1 X3 (u1\_struct\_0 X0)) \Rightarrow ((r4\_lattice3 \\
& X0 X3 X1) \Rightarrow (r1\_lattices X0 X2 X3))))))
\end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned}
& \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices \\
& X0))) \Rightarrow (\forall X1. ((\neg v2\_struct\_0 X1) \wedge ((v10\_lattices X1) \wedge (l3\_lattices \\
& X1))) \Rightarrow ((m2\_nat\_lat X1 X0) \Leftrightarrow ((r1\_tarski (u1\_struct\_0 X1) (u1\_struct\_0 \\
& X0)) \wedge ((u2\_lattices X1 = k1\_realset1 (u2\_lattices X0) (u1\_struct\_0 \\
& X1)) \wedge (u1\_lattices X1 = k1\_realset1 (u1\_lattices X0) (u1\_struct\_0 \\
& X1))))))
\end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned}
& \forall X0. (l3\_lattices X0) \Rightarrow (((\neg v2\_struct\_0 X0) \wedge (v10\_lattices \\
& X0)) \Rightarrow ((\neg v2\_struct\_0 X0) \wedge ((v4\_lattices X0) \wedge ((v5\_lattices X0) \wedge \\
& ((v6\_lattices X0) \wedge ((v7\_lattices X0) \wedge ((v8\_lattices X0) \wedge (v9\_lattices \\
& X0))))))))
\end{aligned} \tag{16}$$

**Theorem 1**

$$\begin{aligned}
& \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge ((v4\_lattice3 \\
& X0) \wedge (l3\_lattices X0)))) \Rightarrow (\forall X1. (m2\_nat\_lat X1 X0) \Rightarrow ((v2\_msualg\_7 \\
& X1 X0) \Rightarrow (v4\_lattice3 X1)))
\end{aligned}$$