

t17\_substlat  
(TMb3MYXzv4Dbchz5vrhPTrhtrgJBv6pR3rA)

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Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k5\_finsub\_1 : \iota \Rightarrow \iota$  be given. Let  $k4\_partfun1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $r1\_tarski : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k3\_substlat : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k4\_substlat : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $r1\_partfun1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k2\_xboole\_0 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v1\_finset\_1 : \iota \Rightarrow o$  be given. Let  $v1\_xboole\_0 : \iota \Rightarrow o$  be given. Let  $v4\_finsub\_1 : \iota \Rightarrow o$  be given. Let  $k1\_substlat : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_zfmisc\_1 : \iota \Rightarrow \iota$  be given. Let  $m2\_subset\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (m1\_subset\_1 X2 (k5\_finsub\_1 \\ & (k4\_partfun1 X0 X1))) \Rightarrow (\forall X3. (X3 \in k3\_substlat X0 X1 X2) \Rightarrow ( \\ & (X3 \in X2) \wedge (\forall X4. ((X4 \in X2) \wedge (r1\_tarski X4 X3)) \Rightarrow (X4 = X3)))) \end{aligned} \quad (1)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (m1\_subset\_1 X2 (k5\_finsub\_1 \\ & (k4\_partfun1 X0 X1))) \Rightarrow (\forall X3. (m1\_subset\_1 X3 (k5\_finsub\_1 \\ & (k4\_partfun1 X0 X1))) \Rightarrow (k4\_substlat X0 X1 X2 X3 = k4\_substlat X0 X1 \\ & X3 X2)) \end{aligned} \quad (2)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (m1\_subset\_1 X2 (k5\_finsub\_1 \\ & (k4\_partfun1 X0 X1))) \Rightarrow (\forall X3. (m1\_subset\_1 X3 (k5\_finsub\_1 \\ & (k4\_partfun1 X0 X1))) \Rightarrow (\forall X4. (m1\_subset\_1 X4 (k4\_partfun1 \\ & X0 X1)) \Rightarrow (\forall X5. (m1\_subset\_1 X5 (k4\_partfun1 X0 X1)) \Rightarrow (((X4 \in \\ & X2) \wedge ((X5 \in X3) \wedge (r1\_partfun1 X4 X5)) \Rightarrow (k2\_xboole\_0 X4 X5 \in k4\_substlat \\ & X0 X1 X2 X3)))))) \end{aligned} \quad (3)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (m1\_subset\_1 X2 (k5\_finsub\_1 \\ & (k4\_partfun1 X0 X1))) \Rightarrow (\forall X3. (m1\_subset\_1 X3 (k5\_finsub\_1 \\ & (k4\_partfun1 X0 X1))) \Rightarrow (\forall X4. \neg (X4 \in k4\_substlat X0 X1 X2 X3) \wedge \\ & (\forall X5. \forall X6. \neg (X5 \in X2) \wedge ((X6 \in X3) \wedge (X4 = k2\_xboole\_0 X5 \\ & X6)))))) \end{aligned} \quad (4)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(m1\_subset\_1 X2 (k5\_finsub\_1 \\ & (k4\_partfun1 X0 X1))) \Rightarrow (\forall X3.(v1\_finset\_1 X3) \Rightarrow (\neg(X3 \in X2) \wedge \\ & (\forall X4.\neg(r1\_tarski X4 X3) \wedge (X4 \in k3\_substlat X0 X1 X2)))) \end{aligned} \quad (5)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(m1\_subset\_1 X2 (k5\_finsub\_1 \\ & (k4\_partfun1 X0 X1))) \Rightarrow (\forall X3.(m1\_subset\_1 X3 (k5\_finsub\_1 \\ & (k4\_partfun1 X0 X1))) \Rightarrow (\forall X4.(v1\_finset\_1 X4) \Rightarrow (\neg(X4 \in k4\_substlat \\ & X0 X1 X2 X3) \wedge (\forall X5.(v1\_finset\_1 X5) \Rightarrow (\neg(r1\_tarski X5 X4) \wedge \\ & (X5 \in k4\_substlat X0 X1 (k3\_substlat X0 X1 X2) X3)))))) \end{aligned} \quad (6)$$

Assume the following.

$$\forall X0.(\neg v1\_xboole\_0 (k5\_finsub\_1 X0)) \wedge (v4\_finsub\_1 (k5\_finsub\_1 X0)) \quad (7)$$

Assume the following.

$$\forall X0.\forall X1.\neg v1\_xboole\_0 (k1\_substlat X0 X1) \quad (8)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.((\neg v1\_xboole\_0 X0) \wedge ((\neg v1\_xboole\_0 X1) \wedge \\ & (m1\_subset\_1 X1 (k1\_zfmisc\_1 X0)))) \Rightarrow (\forall X2.(m2\_subset\_1 \\ & X2 X0 X1) \Rightarrow (m1\_subset\_1 X2 X0)) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.\forall X3.((m1\_subset\_1 X2 \\ & (k5\_finsub\_1 (k4\_partfun1 X0 X1))) \wedge (m1\_subset\_1 X3 (k5\_finsub\_1 \\ & (k4\_partfun1 X0 X1)))) \Rightarrow (m1\_subset\_1 (k4\_substlat X0 X1 X2 X3) ( \\ & k5\_finsub\_1 (k4\_partfun1 X0 X1))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(m1\_subset\_1 X2 (k5\_finsub\_1 \\ & (k4\_partfun1 X0 X1))) \Rightarrow (m2\_subset\_1 (k3\_substlat X0 X1 X2) (k5\_finsub\_1 \\ & (k4\_partfun1 X0 X1)) (k1\_substlat X0 X1)) \end{aligned} \quad (11)$$

Assume the following.

$$\forall X0.\forall X1.m1\_subset\_1 (k1\_substlat X0 X1) (k1\_zfmisc\_1 (k5\_finsub\_1 (k4\_partfun1 X0 X1))) \quad (12)$$

Assume the following.

$$\forall X0.\forall X1.(r1\_tarski X0 X1) \Leftrightarrow (\forall X2.(X2 \in X0) \Rightarrow (X2 \in X1)) \quad (13)$$

Assume the following.

$$\begin{aligned}
& \forall X0.\forall X1.\forall X2.(m1\_subset\_1 X2 (k5\_finsub\_1 \\
& (k4\_partfun1 X0 X1))) \Rightarrow (\forall X3.(m1\_subset\_1 X3 (k5\_finsub\_1 \\
& (k4\_partfun1 X0 X1))) \Rightarrow (k4\_substlat X0 X1 X2 X3 = ReplSep2 (toset \\
& (\lambda X4 : \iota.m1\_subset\_1 X4 (k4\_partfun1 X0 X1))) (\lambda X4 : \iota. \\
& toset (\lambda X5 : \iota.m1\_subset\_1 X5 (k4\_partfun1 X0 X1))) (\lambda X4 : \\
& \iota.\lambda X5 : \iota.(X4 \in X2) \wedge ((X5 \in X3) \wedge (r1\_partfun1 X4 X5))) (\lambda X4 : \\
& \iota.\lambda X5 : \iota.k2\_xboole\_0 X4 X5)))
\end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned}
& \forall X0.\forall X1.\forall X2.(m1\_subset\_1 X2 (k5\_finsub\_1 \\
& (k4\_partfun1 X0 X1))) \Rightarrow (k3\_substlat X0 X1 X2 = ReplSep (toset (\lambda X3 : \\
& \iota.m1\_subset\_1 X3 (k4\_partfun1 X0 X1))) (\lambda X3 : \iota.(v1\_finset\_1 \\
& X3) \wedge (\forall X4.(m1\_subset\_1 X4 (k4\_partfun1 X0 X1)) \Rightarrow (((X4 \in X2) \wedge \\
& (r1\_tarski X4 X3)) \Leftrightarrow (X4 = X3)))) (\lambda X3 : \iota.X3))
\end{aligned} \tag{15}$$

**Theorem 1**

$$\begin{aligned}
& \forall X0.\forall X1.\forall X2.(m1\_subset\_1 X2 (k5\_finsub\_1 \\
& (k4\_partfun1 X0 X1))) \Rightarrow (\forall X3.(m1\_subset\_1 X3 (k5\_finsub\_1 \\
& (k4\_partfun1 X0 X1))) \Rightarrow (r1\_tarski (k3\_substlat X0 X1 (k4\_substlat \\
& X0 X1 X2 X3)) (k4\_substlat X0 X1 (k3\_substlat X0 X1 X2) X3)))
\end{aligned}$$