

t18\_circcmb2 (TMJca-  
NurH4PYbVU7zwMR4yR26XZiFfQNwNd)

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Let  $v2\_struct\_0 : \iota \Rightarrow o$  be given. Let  $v11\_struct\_0 : \iota \Rightarrow o$  be given. Let  $v2\_msafree2 : \iota \Rightarrow o$  be given. Let  $l1\_msualg\_1 : \iota \Rightarrow o$  be given. Let  $r1\_xboole\_0 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k2\_msafree2 : \iota \Rightarrow \iota$  be given. Let  $k3\_msafree2 : \iota \Rightarrow \iota$  be given. Let  $k2\_circcomb : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v4\_msualg\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v4\_msafree2 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $l3\_msualg\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $r2\_circcomb : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k3\_circcomb : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k4\_card\_3 : \iota \Rightarrow \iota$  be given. Let  $u3\_msualg\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k5\_relat\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $u1\_struct\_0 : \iota \Rightarrow \iota$  be given. Let  $v1\_circuit2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v7\_ordinal1 : \iota \Rightarrow o$  be given. Let  $k5\_facirc\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k6\_circuit2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_nat\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $np\_1 : \iota$  be given. Let  $k6\_numbers : \iota$  be given. Assume the following.

$$\begin{aligned}
 & \forall X0.((\neg v2\_struct\_0 X0) \wedge ((\neg v11\_struct\_0 X0) \wedge ((v2\_msafree2 \\
 & \quad X0) \wedge (l1\_msualg\_1 X0)))) \Rightarrow (\forall X1.((v4\_msualg\_1 X1 X0) \wedge (( \\
 & \quad v4\_msafree2 X1 X0) \wedge (l3\_msualg\_1 X1 X0))) \Rightarrow (\forall X2.(m1\_subset\_1 \\
 & \quad X2 (k4\_card\_3 (u3\_msualg\_1 X0 X1))) \Rightarrow ((v1\_circuit2 X2 X0 X1) \Rightarrow (\forall X3. \\
 & \quad (v7\_ordinal1 X3) \Rightarrow (k5\_facirc\_1 X0 X1 X2 X3 = X2))))))
 \end{aligned} \tag{1}$$

Assume the following.

$$\begin{aligned}
& \forall X0.((\neg v2\_struct\_0 X0) \wedge ((\neg v11\_struct\_0 X0) \wedge ((v2\_msafree2 \\
& \quad X0) \wedge (l1\_msualg\_1 X0)))) \Rightarrow (\forall X1.((\neg v2\_struct\_0 X1) \wedge ((\neg \\
& \quad v11\_struct\_0 X1) \wedge ((v2\_msafree2 X1) \wedge (l1\_msualg\_1 X1)))) \Rightarrow (\forall X2. \\
& \quad ((\neg v2\_struct\_0 X2) \wedge ((\neg v11\_struct\_0 X2) \wedge ((v2\_msafree2 X2) \wedge ( \\
& \quad l1\_msualg\_1 X2)))) \Rightarrow (((r1\_xboole\_0 (k2\_msafree2 X0) (k3\_msafree2 \\
& \quad X1)) \wedge (X2 = k2\_circcomb X0 X1)) \Rightarrow (\forall X3.((v4\_msualg\_1 X3 X0) \wedge \\
& \quad ((v4\_msafree2 X3 X0) \wedge (l3\_msualg\_1 X3 X0))) \Rightarrow (\forall X4.((v4\_msualg\_1 \\
& \quad X4 X1) \wedge ((v4\_msafree2 X4 X1) \wedge (l3\_msualg\_1 X4 X1))) \Rightarrow (\forall X5. \\
& \quad ((v4\_msualg\_1 X5 X2) \wedge ((v4\_msafree2 X5 X2) \wedge (l3\_msualg\_1 X5 X2))) \Rightarrow \\
& \quad (((r2\_circcomb X0 X1 X3 X4) \wedge (X5 = k3\_circcomb X0 X1 X3 X4)) \Rightarrow (\forall X6. \\
& \quad (m1\_subset\_1 X6 (k4\_card\_3 (u3\_msualg\_1 X2 X5))) \Rightarrow (\forall X7. \\
& \quad (m1\_subset\_1 X7 (k4\_card\_3 (u3\_msualg\_1 X0 X3))) \Rightarrow (((X7 = k5\_relat\_1 \\
& \quad X6 (u1\_struct\_0 X0)) \wedge (v1\_circuit2 X7 X0 X3)) \Rightarrow (\forall X8.(m1\_subset\_1 \\
& \quad X8 (k4\_card\_3 (u3\_msualg\_1 X1 X4)) \Rightarrow ((X8 = k5\_relat\_1 X6 (u1\_struct\_0 \\
& \quad X1)) \Rightarrow (k5\_relat\_1 (k6\_circuit2 X2 X5 X6) (u1\_struct\_0 X1) = k6\_circuit2 \\
& \quad X1 X4 X8))))))))))))) \\
& \hspace{15em} (2)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall X0.((\neg v2\_struct\_0 X0) \wedge ((\neg v11\_struct\_0 X0) \wedge ((v2\_msafree2 \\
& \quad X0) \wedge (l1\_msualg\_1 X0)))) \Rightarrow (\forall X1.((\neg v2\_struct\_0 X1) \wedge ((\neg \\
& \quad v11\_struct\_0 X1) \wedge ((v2\_msafree2 X1) \wedge (l1\_msualg\_1 X1)))) \Rightarrow (\forall X2. \\
& \quad ((\neg v2\_struct\_0 X2) \wedge ((\neg v11\_struct\_0 X2) \wedge ((v2\_msafree2 X2) \wedge ( \\
& \quad l1\_msualg\_1 X2)))) \Rightarrow (((r1\_xboole\_0 (k2\_msafree2 X0) (k3\_msafree2 \\
& \quad X1)) \wedge (X2 = k2\_circcomb X0 X1)) \Rightarrow (\forall X3.((v4\_msualg\_1 X3 X0) \wedge \\
& \quad ((v4\_msafree2 X3 X0) \wedge (l3\_msualg\_1 X3 X0))) \Rightarrow (\forall X4.((v4\_msualg\_1 \\
& \quad X4 X1) \wedge ((v4\_msafree2 X4 X1) \wedge (l3\_msualg\_1 X4 X1))) \Rightarrow (\forall X5. \\
& \quad ((v4\_msualg\_1 X5 X2) \wedge ((v4\_msafree2 X5 X2) \wedge (l3\_msualg\_1 X5 X2))) \Rightarrow \\
& \quad (((r2\_circcomb X0 X1 X3 X4) \wedge (X5 = k3\_circcomb X0 X1 X3 X4)) \Rightarrow (\forall X6. \\
& \quad (m1\_subset\_1 X6 (k4\_card\_3 (u3\_msualg\_1 X2 X5))) \Rightarrow (\forall X7. \\
& \quad (m1\_subset\_1 X7 (k4\_card\_3 (u3\_msualg\_1 X0 X3))) \Rightarrow ((X7 = k5\_relat\_1 \\
& \quad X6 (u1\_struct\_0 X0)) \Rightarrow (\forall X8.(v7\_ordinal1 X8) \Rightarrow (k5\_relat\_1 \\
& \quad (k5\_facirc\_1 X2 X5 X6 X8) (u1\_struct\_0 X0) = k5\_facirc\_1 X0 X3 X7 X8))))))))))))) \\
& \hspace{15em} (3)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall X0.((\neg v2\_struct\_0 X0) \wedge ((\neg v11\_struct\_0 X0) \wedge ((v2\_msafree2 \\
& \quad X0) \wedge (l1\_msualg\_1 X0)))) \Rightarrow (\forall X1.((v4\_msualg\_1 X1 X0) \wedge (( \\
& \quad v4\_msafree2 X1 X0) \wedge (l3\_msualg\_1 X1 X0))) \Rightarrow (\forall X2.(m1\_subset\_1 \\
& \quad X2 (k4\_card\_3 (u3\_msualg\_1 X0 X1))) \Rightarrow (\forall X3.(v7\_ordinal1 \\
& \quad X3) \Rightarrow (k5\_facirc\_1 X0 X1 X2 (k1\_nat\_1 X3 np\_1) = k6\_circuit2 X0 X1 \\
& \quad (k5\_facirc\_1 X0 X1 X2 X3)))))) \\
& \hspace{15em} (4)
\end{aligned}$$

Assume the following.

$$\begin{aligned} \forall X0.((\neg v2\_struct\_0 X0) \wedge ((\neg v11\_struct\_0 X0) \wedge ((v2\_msafree2 \\ X0) \wedge (l1\_msualg\_1 X0)))) \Rightarrow (\forall X1.((v4\_msualg\_1 X1 X0) \wedge (( \\ v4\_msafree2 X1 X0) \wedge (l3\_msualg\_1 X1 X0))) \Rightarrow (\forall X2.(m1\_subset\_1 \\ X2 (k4\_card\_3 (u3\_msualg\_1 X0 X1))) \Rightarrow (k5\_facirc\_1 X0 X1 X2 k6\_numbers = \\ X2))) \end{aligned} \quad (5)$$

Assume the following.

$$\forall X0 : \iota \Rightarrow o.((X0 k6\_numbers) \wedge (\forall X1.(v7\_ordinal1 \\ X1) \Rightarrow ((X0 X1) \Rightarrow (X0 (k1\_nat\_1 X1 np\_1)))))) \Rightarrow (\forall X1.(v7\_ordinal1 \\ X1) \Rightarrow (X0 X1)) \quad (6)$$

Assume the following.

$$\begin{aligned} \forall X0.\forall X1.\forall X2.\forall X3.(((\neg v2\_struct\_0 \\ X0) \wedge ((\neg v11\_struct\_0 X0) \wedge ((v2\_msafree2 X0) \wedge (l1\_msualg\_1 X0)))) \wedge \\ (((v4\_msualg\_1 X1 X0) \wedge ((v4\_msafree2 X1 X0) \wedge (l3\_msualg\_1 X1 X0))) \wedge \\ ((m1\_subset\_1 X2 (k4\_card\_3 (u3\_msualg\_1 X0 X1))) \wedge (v7\_ordinal1 \\ X3)))) \Rightarrow (m1\_subset\_1 (k5\_facirc\_1 X0 X1 X2 X3) (k4\_card\_3 (u3\_msualg\_1 \\ X0 X1))) \end{aligned} \quad (7)$$

**Theorem 1**

$$\begin{aligned} \forall X0.((\neg v2\_struct\_0 X0) \wedge ((\neg v11\_struct\_0 X0) \wedge ((v2\_msafree2 \\ X0) \wedge (l1\_msualg\_1 X0)))) \Rightarrow (\forall X1.((\neg v2\_struct\_0 X1) \wedge ((\neg \\ v11\_struct\_0 X1) \wedge ((v2\_msafree2 X1) \wedge (l1\_msualg\_1 X1)))) \Rightarrow (\forall X2. \\ ((\neg v2\_struct\_0 X2) \wedge ((\neg v11\_struct\_0 X2) \wedge ((v2\_msafree2 X2) \wedge ( \\ l1\_msualg\_1 X2)))) \Rightarrow (((r1\_xboole\_0 (k2\_msafree2 X0) (k3\_msafree2 \\ X1)) \wedge (X2 = k2\_circcomb X0 X1)) \Rightarrow (\forall X3.((v4\_msualg\_1 X3 X0) \wedge \\ ((v4\_msafree2 X3 X0) \wedge (l3\_msualg\_1 X3 X0))) \Rightarrow (\forall X4.((v4\_msualg\_1 \\ X4 X1) \wedge ((v4\_msafree2 X4 X1) \wedge (l3\_msualg\_1 X4 X1))) \Rightarrow (\forall X5. \\ ((v4\_msualg\_1 X5 X2) \wedge ((v4\_msafree2 X5 X2) \wedge (l3\_msualg\_1 X5 X2))) \Rightarrow \\ (((r2\_circcomb X0 X1 X3 X4) \wedge (X5 = k3\_circcomb X0 X1 X3 X4)) \Rightarrow (\forall X6. \\ (m1\_subset\_1 X6 (k4\_card\_3 (u3\_msualg\_1 X0 X3))) \Rightarrow (\forall X7. \\ (m1\_subset\_1 X7 (k4\_card\_3 (u3\_msualg\_1 X1 X4))) \Rightarrow (\forall X8. \\ (m1\_subset\_1 X8 (k4\_card\_3 (u3\_msualg\_1 X2 X5))) \Rightarrow (((X6 = k5\_relat\_1 \\ X8 (u1\_struct\_0 X0)) \wedge ((X7 = k5\_relat\_1 X8 (u1\_struct\_0 X1)) \wedge (v1\_circuit2 \\ X6 X0 X3))) \Rightarrow (\forall X9.(v7\_ordinal1 X9) \Rightarrow (k5\_relat\_1 (k5\_facirc\_1 \\ X2 X5 X8 X9) (u1\_struct\_0 X1) = k5\_facirc\_1 X1 X4 X7 X9)))))))))) \end{aligned}$$