t18_closure1 (TMQxiTAP9icFEZ1jfMj9PH9BfkMF2PtnShP)

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Let $v1\_relat\_1 : \iota \Rightarrow o$ be given. Let $v4\_relat\_1 : \iota \Rightarrow \iota \Rightarrow o$ be given. Let $v1\_funct\_1 : \iota \Rightarrow o$ be given. Let $v1\_partfun1 : \iota \Rightarrow \iota \Rightarrow o$ be given. Let $m2\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$ be given. Let $k5\_mssubfam : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Let $m1\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$ be given. Let $v2\_closure1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$ be given. Let $r2\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$ be given. Let $k16\_pralg\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Let $k3\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Let $k2\_closure1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Let $r6\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$ be given. Let $k1\_mboolean : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Let $r1\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$ be given. Let $v2\_relat\_1 : \iota \Rightarrow o$ be given. Let $k15\_pralg\_1 : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Let $v1\_funcop\_1 : \iota \Rightarrow o$ be given. Let $v1\_mssubfam : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$ be given. Let $v2\_mssubfam : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$ be given. Let $v3\_mssubfam : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$ be given. Let $v4\_mssubfam : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$ be given. Let $v5\_mssubfam : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$ be given. Let $v6\_mssubfam : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$ be given. Let $m3\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$ be given. Assume the following.

$$\forall X0.\forall X1.((v1\_relat\_1\ X1)\wedge((v4\_relat\_1\ X1\ X0)\wedge((v1\_funct\_1\ X1)\wedge(v1\_partfun1\ X1\ X0))))\Rightarrow(\forall X2.((v1\_relat\_1\ X2)\wedge((v4\_relat\_1\ X2\ X0)\wedge((v1\_funct\_1\ X2)\wedge(v1\_partfun1\ X2\ X0))))\Rightarrow((r6\_pboole\ X0\ X1\ (k1\_mboolean\ X0\ X2))\Leftrightarrow(\forall X3.((v1\_relat\_1\ X3)\wedge((v4\_relat\_1\ X3\ X0)\wedge((v1\_funct\_1\ X3)\wedge(v1\_partfun1\ X3\ X0))))\Rightarrow((r1\_pboole\ X0\ X3\ X1)\Leftrightarrow(r2\_pboole\ X0\ X3\ X2)))))) \tag{1}$$

Assume the following.

$$\forall X0.\forall X1.((v1\_relat\_1\ X1)\wedge((v4\_relat\_1\ X1\ X0)\wedge((v1\_funct\_1\ X1)\wedge(v1\_partfun1\ X1\ X0))))\Rightarrow(\forall X2.((v1\_relat\_1\ X2)\wedge((v4\_relat\_1\ X2\ X0)\wedge((v1\_funct\_1\ X2)\wedge(v1\_partfun1\ X2\ X0))))\Rightarrow(\forall X3.((v1\_relat\_1\ X3)\wedge((v4\_relat\_1\ X3\ X0)\wedge((v1\_funct\_1\ X3)\wedge(v1\_partfun1\ X3\ X0))))\Rightarrow(((r2\_pboole\ X0\ X1\ X2)\wedge(r2\_pboole\ X0\ X1\ X3))\Rightarrow(r2\_pboole\ X0\ X1\ (k3\_pboole\ X0\ X2\ X3))))) \tag{2}$$

Assume the following.

$$\forall X0.\forall X1.((v1\_relat\_1\ X1)\wedge((v4\_relat\_1\ X1\ X0)\wedge((v1\_funct\_1\ X1)\wedge(v1\_partfun1\ X1\ X0))))\Rightarrow(\forall X2.((v1\_relat\_1\ X2)\wedge((v4\_relat\_1\ X2\ X0)\wedge((v1\_funct\_1\ X2)\wedge(v1\_partfun1\ X2\ X0))))\Rightarrow(r2\_pboole\ X0\ (k3\_pboole\ X0\ X1\ X2)\ X1)) \tag{3}$$

Assume the following.

$$\forall X0.\forall X1.((v1\_relat\_1\ X1)\wedge((v4\_relat\_1\ X1\ X0)\wedge((v1\_funct\_1\ X1)\wedge(v1\_partfun1\ X1\ X0))))\Rightarrow(\forall X2.((v1\_relat\_1\ X2)\wedge((v4\_relat\_1\ X2\ X0)\wedge((v1\_funct\_1\ X2)\wedge(v1\_partfun1\ X2\ X0))))\Rightarrow(\forall X3.((v1\_relat\_1\ X3)\wedge((v4\_relat\_1\ X3\ X0)\wedge((v1\_funct\_1\ X3)\wedge(v1\_partfun1\ X3\ X0))))\Rightarrow(((r2\_pboole\ X0\ X1\ X2)\vee(r2\_pboole\ X0\ X3\ X2))\Rightarrow(r2\_pboole\ X0\ (k3\_pboole\ X0\ X1\ X3)\ X2)))) \tag{4}$$

Assume the following.

$$\forall X0.\forall X1.((v1\_relat\_1\ X1)\wedge((v4\_relat\_1\ X1\ X0)\wedge((v1\_funct\_1\ X1)\wedge(v1\_partfun1\ X1\ X0))))\Rightarrow(\forall X2.((v1\_relat\_1\ X2)\wedge((v2\_relat\_1\ X2)\wedge((v4\_relat\_1\ X2\ X0)\wedge((v1\_funct\_1\ X2)\wedge(v1\_partfun1\ X2\ X0)))))\Rightarrow((m1\_pboole\ X1\ X0\ X2)\Rightarrow(r1\_pboole\ X0\ X1\ X2))) \tag{5}$$

Assume the following.

$$\forall X0.\forall X1.((v1\_relat\_1\ X1)\wedge((v4\_relat\_1\ X1\ X0)\wedge((v1\_funct\_1\ X1)\wedge(v1\_partfun1\ X1\ X0))))\Rightarrow(\forall X2.((v1\_relat\_1\ X2)\wedge((v4\_relat\_1\ X2\ X0)\wedge((v1\_funct\_1\ X2)\wedge(v1\_partfun1\ X2\ X0))))\Rightarrow((r1\_pboole\ X0\ X1\ X2)\Rightarrow(m1\_pboole\ X1\ X0\ X2))) \tag{6}$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((v1\_relat\_1\ X1)\wedge((v4\_relat\_1\ X1\ X0)\wedge((v1\_funct\_1\ X1)\wedge(v1\_partfun1\ X1\ X0))))\wedge((v1\_relat\_1\ X2)\wedge((v4\_relat\_1\ X2\ X0)\wedge((v1\_funct\_1\ X2)\wedge(v1\_partfun1\ X2\ X0)))))\Rightarrow(r6\_pboole\ X0\ X1\ X1) \tag{7}$$

Assume the following.

$$\forall X0.\forall X1.((v1\_relat\_1\ X1)\wedge((v4\_relat\_1\ X1\ X0)\wedge((v1\_funct\_1\ X1)\wedge(v1\_partfun1\ X1\ X0))))\Rightarrow(k5\_mssubfam\ X0\ X1 = k1\_mboolean\ X0\ X1) \tag{8}$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.\forall X3.(((v1\_relat\_1\ X1)\wedge((v4\_relat\_1\ X1\ X0)\wedge((v1\_funct\_1\ X1)\wedge(v1\_partfun1\ X1\ X0))))\wedge((m2\_pboole\ X2\ X0\ (k5\_mssubfam\ X0\ X1)\ (k5\_mssubfam\ X0\ X1))\wedge(m1\_pboole\ X3\ X0\ (k5\_mssubfam\ X0\ X1))))\Rightarrow(k2\_closure1\ X0\ X1\ X2\ X3 = k15\_pralg\_1\ X2\ X3) \tag{9}$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((v1\_relat\_1\ X1)\wedge((v4\_relat\_1\ X1\ X0)\wedge((v1\_funct\_1\ X1)\wedge((v1\_partfun1\ X1\ X0)\wedge(v1\_funcop\_1\ X1)))))\wedge((v1\_relat\_1\ X2)\wedge((v4\_relat\_1\ X2\ X0)\wedge((v1\_funct\_1\ X2)\wedge(v1\_partfun1\ X2\ X0)))))\Rightarrow(k16\_pralg\_1\ X0\ X1\ X2 = k15\_pralg\_1\ X1\ X2) \tag{10}$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((v1\_relat\_1\ X1)\wedge((v4\_relat\_1\ X1\ X0)\wedge((v1\_funct\_1\ X1)\wedge(v1\_partfun1\ X1\ X0))))\wedge((v1\_relat\_1\ X2)\wedge((v4\_relat\_1\ X2\ X0)\wedge((v1\_funct\_1\ X2)\wedge(v1\_partfun1\ X2\ X0)))))\Rightarrow(\forall X3.(m2\_pboole\ X3\ X0\ X1\ X2)\Rightarrow((v1\_relat\_1\ X3)\wedge((v4\_relat\_1\ X3\ X0)\wedge((v1\_funct\_1\ X3)\wedge(v1\_partfun1\ X3\ X0)))))) \tag{11}$$

Assume the following.

$$\forall X0.\forall X1.((v1\_relat\_1\ X1)\wedge((v4\_relat\_1\ X1\ X0)\wedge((v1\_funct\_1\ X1)\wedge(v1\_partfun1\ X1\ X0))))\Rightarrow(\forall X2.(m1\_pboole\ X2\ X0\ X1)\Rightarrow((v1\_relat\_1\ X2)\wedge((v4\_relat\_1\ X2\ X0)\wedge((v1\_funct\_1\ X2)\wedge(v1\_partfun1\ X2\ X0))))) \tag{12}$$

Assume the following.

$$\forall X0.\forall X1.((v1\_relat\_1\ X1)\wedge((v4\_relat\_1\ X1\ X0)\wedge((v1\_funct\_1\ X1)\wedge(v1\_partfun1\ X1\ X0))))\Rightarrow((v1\_mssubfam\ (k5\_mssubfam\ X0\ X1)\ X0\ X1)\wedge((v2\_mssubfam\ (k5\_mssubfam\ X0\ X1)\ X0\ X1)\wedge((v3\_mssubfam\ (k5\_mssubfam\ X0\ X1)\ X0\ X1)\wedge((v4\_mssubfam\ (k5\_mssubfam\ X0\ X1)\ X0\ X1)\wedge((v5\_mssubfam\ (k5\_mssubfam\ X0\ X1)\ X0\ X1)\wedge((v6\_mssubfam\ (k5\_mssubfam\ X0\ X1)\ X0\ X1)\wedge(m3\_pboole\ (k5\_mssubfam\ X0\ X1)\ X0\ (k1\_mboolean\ X0\ X1)))))))) \tag{13}$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((v1\_relat\_1\ X1)\wedge((v4\_relat\_1\ X1\ X0)\wedge((v1\_funct\_1\ X1)\wedge(v1\_partfun1\ X1\ X0))))\wedge((v1\_relat\_1\ X2)\wedge((v4\_relat\_1\ X2\ X0)\wedge((v1\_funct\_1\ X2)\wedge(v1\_partfun1\ X2\ X0)))))\Rightarrow((v1\_relat\_1\ (k3\_pboole\ X0\ X1\ X2))\wedge((v4\_relat\_1\ (k3\_pboole\ X0\ X1\ X2)\ X0)\wedge((v1\_funct\_1\ (k3\_pboole\ X0\ X1\ X2))\wedge(v1\_partfun1\ (k3\_pboole\ X0\ X1\ X2)\ X0)))) \tag{14}$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.\forall X3.(((v1\_relat\_1\ X1)\wedge ((v4\_relat\_1\ X1\ X0)\wedge((v1\_funct\_1\ X1)\wedge(v1\_partfun1\ X1\ X0))))\wedge ((m2\_pboole\ X2\ X0\ (k5\_mssubfam\ X0\ X1)\ (k5\_mssubfam\ X0\ X1))\wedge(m1\_pboole\ X3\ X0\ (k5\_mssubfam\ X0\ X1))))\Rightarrow(m1\_pboole\ (k2\_closure1\ X0\ X1\ X2\ X3)\ X0\ (k5\_mssubfam\ X0\ X1)) \tag{15}$$

Assume the following.

$$\forall X0.\forall X1.((v1\_relat\_1\ X1)\wedge((v4\_relat\_1\ X1\ X0)\wedge((v1\_funct\_1\ X1)\wedge(v1\_partfun1\ X1\ X0))))\Rightarrow((v1\_relat\_1\ (k1\_mboolean\ X0\ X1))\wedge((v4\_relat\_1\ (k1\_mboolean\ X0\ X1)\ X0)\wedge((v1\_funct\_1\ (k1\_mboolean\ X0\ X1))\wedge(v1\_partfun1\ (k1\_mboolean\ X0\ X1)\ X0)))) \tag{16}$$

Assume the following.

$$\forall X0.\forall X1.((v1\_relat\_1\ X1)\wedge((v4\_relat\_1\ X1\ X0)\wedge((v1\_funct\_1\ X1)\wedge(v1\_partfun1\ X1\ X0))))\Rightarrow(\forall X2.(m2\_pboole\ X2\ X0\ (k5\_mssubfam\ X0\ X1)\ (k5\_mssubfam\ X0\ X1))\Rightarrow((v2\_closure1\ X2\ X0\ X1)\Leftrightarrow(\forall X3.(m1\_pboole\ X3\ X0\ (k5\_mssubfam\ X0\ X1))\Rightarrow(\forall X4.(m1\_pboole\ X4\ X0\ (k5\_mssubfam\ X0\ X1))\Rightarrow((r2\_pboole\ X0\ X3\ X4)\Rightarrow(r2\_pboole\ X0\ (k2\_closure1\ X0\ X1\ X2\ X3)\ (k2\_closure1\ X0\ X1\ X2\ X4))))))) \tag{17}$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((v1\_relat\_1\ X1)\wedge((v4\_relat\_1\ X1\ X0)\wedge((v1\_funct\_1\ X1)\wedge(v1\_partfun1\ X1\ X0))))\wedge((v1\_relat\_1\ X2)\wedge((v4\_relat\_1\ X2\ X0)\wedge((v1\_funct\_1\ X2)\wedge(v1\_partfun1\ X2\ X0)))))\Rightarrow(k3\_pboole\ X0\ X1\ X2 = k3\_pboole\ X0\ X2\ X1) \tag{18}$$

Assume the following.

$$\forall X0.\forall X1.((v1\_relat\_1\ X1)\wedge((v4\_relat\_1\ X1\ X0)\wedge((v1\_funct\_1\ X1)\wedge(v1\_partfun1\ X1\ X0))))\Rightarrow(\forall X2.(m3\_pboole\ X2\ X0\ (k1\_mboolean\ X0\ X1))\Rightarrow((v5\_mssubfam\ X2\ X0\ X1)\Rightarrow(v2\_relat\_1\ X2))) \tag{19}$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((v1\_relat\_1\ X1)\wedge((v4\_relat\_1\ X1\ X0)\wedge((v1\_funct\_1\ X1)\wedge(v1\_partfun1\ X1\ X0))))\wedge((v1\_relat\_1\ X2)\wedge((v4\_relat\_1\ X2\ X0)\wedge((v1\_funct\_1\ X2)\wedge(v1\_partfun1\ X2\ X0)))))\Rightarrow(\forall X3.(m2\_pboole\ X3\ X0\ X1\ X2)\Rightarrow(v1\_funcop\_1\ X3)) \tag{20}$$

Theorem 1

$$\forall X0.\forall X1.((v1\_relat\_1\ X1)\wedge((v4\_relat\_1\ X1\ X0)\wedge((v1\_funct\_1\ X1)\wedge(v1\_partfun1\ X1\ X0))))\Rightarrow(\forall X2.(m2\_pboole\ X2\ X0\ (k5\_mssubfam\ X0\ X1)\ (k5\_mssubfam\ X0\ X1))\Rightarrow(\forall X3.(m1\_pboole\ X3\ X0\ (k5\_mssubfam\ X0\ X1))\Rightarrow(\forall X4.(m1\_pboole\ X4\ X0\ (k5\_mssubfam\ X0\ X1))\Rightarrow((v2\_closure1\ X2\ X0\ X1)\Rightarrow(r2\_pboole\ X0\ (k16\_pralg\_1\ X0\ X2\ (k3\_pboole\ X0\ X3\ X4))\ (k3\_pboole\ X0\ (k2\_closure1\ X0\ X1\ X2\ X3)\ (k2\_closure1\ X0\ X1\ X2\ X4)))))))$$