

## t20\_lattice2 (TMGhvnDXqLGWBKAVYvBogin- Npir8v7YYaRx)

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Let  $v2\_struct\_0 : \iota \Rightarrow o$  be given. Let  $v10\_lattices : \iota \Rightarrow o$  be given. Let  $l3\_lattices : \iota \Rightarrow o$  be given. Let  $r6\_binop\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $u1\_struct\_0 : \iota \Rightarrow \iota$  be given. Let  $u2\_lattices : \iota \Rightarrow \iota$  be given. Let  $v1\_xboole\_0 : \iota \Rightarrow o$  be given. Let  $v1\_funct\_1 : \iota \Rightarrow o$  be given. Let  $v1\_funct\_2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k2\_zfmisc\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k1\_zfmisc\_1 : \iota \Rightarrow \iota$  be given. Let  $v1\_binop\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k3\_binop\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k5\_binop\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_binop\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v4\_lattices : \iota \Rightarrow o$  be given. Let  $l2\_lattices : \iota \Rightarrow o$  be given. Let  $k3\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v6\_lattices : \iota \Rightarrow o$  be given. Let  $v8\_lattices : \iota \Rightarrow o$  be given. Let  $v9\_lattices : \iota \Rightarrow o$  be given. Let  $l1\_struct\_0 : \iota \Rightarrow o$  be given. Let  $l1\_lattices : \iota \Rightarrow o$  be given. Let  $v5\_lattices : \iota \Rightarrow o$  be given. Let  $v7\_lattices : \iota \Rightarrow o$  be given. Assume the following.

$$\begin{aligned}
& \forall X0. (\neg v1\_xboole\_0 X0) \Rightarrow (\forall X1. ((v1\_funct\_1 X1) \wedge \\
& (v1\_funct\_2 X1 (k2\_zfmisc\_1 X0 X0) X0) \wedge (m1\_subset\_1 X1 (k1\_zfmisc\_1 \\
& (k2\_zfmisc\_1 (k2\_zfmisc\_1 X0 X0) X0)))) \Rightarrow (\forall X2. ((v1\_funct\_1 \\
& X2) \wedge ((v1\_funct\_2 X2 (k2\_zfmisc\_1 X0 X0) X0) \wedge (m1\_subset\_1 X2 (k1\_zfmisc\_1 \\
& (k2\_zfmisc\_1 (k2\_zfmisc\_1 X0 X0) X0)))) \Rightarrow ((v1\_binop\_1 X2 X0) \Rightarrow \\
& ((r6\_binop\_1 X0 X2 X1) \Leftrightarrow (\forall X3. (m1\_subset\_1 X3 X0) \Rightarrow (\forall X4. \\
& (m1\_subset\_1 X4 X0) \Rightarrow (\forall X5. (m1\_subset\_1 X5 X0) \Rightarrow (k3\_binop\_1 \\
& X0 X2 X3 (k3\_binop\_1 X0 X1 X4 X5) = k3\_binop\_1 X0 X1 (k3\_binop\_1 X0 X2 \\
& X3 X4) (k3\_binop\_1 X0 X2 X3 X5))))))))))
\end{aligned} \tag{1}$$

Assume the following.

$$\begin{aligned}
& \forall X0. \forall X1. \forall X2. \forall X3. (((v1\_funct\_1 X1) \wedge \\
& ((v1\_funct\_2 X1 (k2\_zfmisc\_1 X0 X0) X0) \wedge (m1\_subset\_1 X1 (k1\_zfmisc\_1 \\
& (k2\_zfmisc\_1 (k2\_zfmisc\_1 X0 X0) X0)))) \wedge ((m1\_subset\_1 X2 X0) \wedge \\
& (m1\_subset\_1 X3 X0))) \Rightarrow (k5\_binop\_1 X0 X1 X2 X3 = k1\_binop\_1 X1 X2 X3)
\end{aligned} \tag{2}$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge(v4\_lattices X0)\wedge(l2\_lattices X0))\wedge((m1\_subset\_1 X1 (u1\_struct\_0 X0))\wedge(m1\_subset\_1 X2 (u1\_struct\_0 X0))))\Rightarrow(k3\_lattices X0 X1 X2 = k1\_lattices X0 X1 X2) \quad (3)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.\forall X3.(((v1\_funct\_1 X1)\wedge((v1\_funct\_2 X1 (k2\_zfmisc\_1 X0 X0) X0)\wedge(m1\_subset\_1 X1 (k1\_zfmisc\_1 (k2\_zfmisc\_1 (k2\_zfmisc\_1 X0 X0) X0))))\wedge((m1\_subset\_1 X2 X0)\wedge(m1\_subset\_1 X3 X0))))\Rightarrow(k3\_binop\_1 X0 X1 X2 X3 = k1\_binop\_1 X1 X2 X3) \quad (4)$$

Assume the following.

$$\forall X0.\forall X1.(((\neg v2\_struct\_0 X0)\wedge(v6\_lattices X0)\wedge((v8\_lattices X0)\wedge(v9\_lattices X0)\wedge(l3\_lattices X0))))\wedge(m1\_subset\_1 X1 (u1\_struct\_0 X0))\Rightarrow(k1\_lattices X0 X1 X1 = X1) \quad (5)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0)\wedge(v4\_lattices X0)\wedge(l2\_lattices X0))\Rightarrow((v1\_funct\_1 (u2\_lattices X0))\wedge((v1\_funct\_2 (u2\_lattices X0) (k2\_zfmisc\_1 (u1\_struct\_0 X0) (u1\_struct\_0 X0)) (u1\_struct\_0 X0))\wedge(v1\_binop\_1 (u2\_lattices X0) (u1\_struct\_0 X0)))) \quad (6)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0)\wedge(l1\_struct\_0 X0))\Rightarrow(\neg v1\_xboole\_0 (u1\_struct\_0 X0)) \quad (7)$$

Assume the following.

$$\forall X0.(l2\_lattices X0)\Rightarrow(((v1\_funct\_1 (u2\_lattices X0))\wedge((v1\_funct\_2 (u2\_lattices X0) (k2\_zfmisc\_1 (u1\_struct\_0 X0) (u1\_struct\_0 X0)) (u1\_struct\_0 X0))\wedge(m1\_subset\_1 (u2\_lattices X0) (k1\_zfmisc\_1 (k2\_zfmisc\_1 (k2\_zfmisc\_1 (u1\_struct\_0 X0) (u1\_struct\_0 X0)) (u1\_struct\_0 X0)))))) \quad (8)$$

Assume the following.

$$\forall X0.(l3\_lattices X0)\Rightarrow((l1\_lattices X0)\wedge(l2\_lattices X0)) \quad (9)$$

Assume the following.

$$\forall X0.(l2\_lattices X0)\Rightarrow(l1\_struct\_0 X0) \quad (10)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.\forall X3.(((v1\_funct\_1 X1)\wedge \\ & ((v1\_funct\_2 X1 (k2\_zfmisc\_1 X0 X0) X0)\wedge(m1\_subset\_1 X1 (k1\_zfmisc\_1 \\ & (k2\_zfmisc\_1 (k2\_zfmisc\_1 X0 X0) X0))))\wedge((m1\_subset\_1 X2 X0)\wedge \\ & (m1\_subset\_1 X3 X0)))\Rightarrow(m1\_subset\_1 (k3\_binop\_1 X0 X1 X2 X3) X0) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} & \forall X0.((\neg v2\_struct\_0 X0)\wedge(l2\_lattices X0))\Rightarrow((v5\_lattices \\ & X0)\Leftrightarrow(\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 X0))\Rightarrow(\forall X2. \\ & (m1\_subset\_1 X2 (u1\_struct\_0 X0))\Rightarrow(\forall X3.(m1\_subset\_1 X3 \\ & (u1\_struct\_0 X0))\Rightarrow(k1\_lattices X0 X1 (k1\_lattices X0 X2 X3) = k1\_lattices \\ & X0 (k1\_lattices X0 X1 X2) X3)))))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} & \forall X0.((\neg v2\_struct\_0 X0)\wedge(l2\_lattices X0))\Rightarrow(\forall X1. \\ & (m1\_subset\_1 X1 (u1\_struct\_0 X0))\Rightarrow(\forall X2.(m1\_subset\_1 X2 \\ & (u1\_struct\_0 X0))\Rightarrow(k1\_lattices X0 X1 X2 = k5\_binop\_1 (u1\_struct\_0 \\ & X0) (u2\_lattices X0) X1 X2))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge((v4\_lattices \\ & X0)\wedge(l2\_lattices X0)))\wedge((m1\_subset\_1 X1 (u1\_struct\_0 X0))\wedge( \\ & m1\_subset\_1 X2 (u1\_struct\_0 X0))))\Rightarrow(k3\_lattices X0 X1 X2 = k3\_lattices \\ & X0 X2 X1) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} & \forall X0.(l3\_lattices X0)\Rightarrow(((\neg v2\_struct\_0 X0)\wedge(v10\_lattices \\ & X0))\Rightarrow((\neg v2\_struct\_0 X0)\wedge((v4\_lattices X0)\wedge((v5\_lattices X0)\wedge \\ & ((v6\_lattices X0)\wedge((v7\_lattices X0)\wedge((v8\_lattices X0)\wedge(v9\_lattices \\ & X0)))))))))) \end{aligned} \quad (15)$$

**Theorem 1**

$$\begin{aligned} & \forall X0.((\neg v2\_struct\_0 X0)\wedge((v10\_lattices X0)\wedge(l3\_lattices \\ & X0)))\Rightarrow(r6\_binop\_1 (u1\_struct\_0 X0) (u2\_lattices X0) (u2\_lattices \\ & X0)) \end{aligned}$$