

# t20\_lattice4 (TMdDyPT- DBF4TUqpYzHf3E4JN3vJgoVDtE8n)

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Let  $v2\_struct\_0 : \iota \Rightarrow o$  be given. Let  $v10\_lattices : \iota \Rightarrow o$  be given. Let  $v14\_lattices : \iota \Rightarrow o$  be given. Let  $l3\_lattices : \iota \Rightarrow o$  be given. Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k5\_finsub\_1 : \iota \Rightarrow \iota$  be given. Let  $u1\_struct\_0 : \iota \Rightarrow \iota$  be given. Let  $v1\_funct\_1 : \iota \Rightarrow o$  be given. Let  $v1\_funct\_2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k1\_zfmisc\_1 : \iota \Rightarrow \iota$  be given. Let  $k2\_zfmisc\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k3\_lattice2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k5\_setwiseo : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k2\_setwiseo : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k4\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k3\_funct\_2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v1\_xboole\_0 : \iota \Rightarrow o$  be given. Let  $v3\_binop\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v1\_binop\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v2\_binop\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v1\_setwiseo : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k7\_setwiseo : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k5\_binop\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v6\_lattices : \iota \Rightarrow o$  be given. Let  $l1\_lattices : \iota \Rightarrow o$  be given. Let  $k2\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v7\_lattices : \iota \Rightarrow o$  be given. Let  $u1\_lattices : \iota \Rightarrow \iota$  be given. Let  $l1\_struct\_0 : \iota \Rightarrow o$  be given. Let  $l2\_lattices : \iota \Rightarrow o$  be given. Let  $v4\_lattices : \iota \Rightarrow o$  be given. Let  $v5\_lattices : \iota \Rightarrow o$  be given. Let  $v8\_lattices : \iota \Rightarrow o$  be given. Let  $v9\_lattices : \iota \Rightarrow o$  be given. Assume the following.

$$\begin{aligned}
& \forall X0. (\neg v1\_xboole\_0 X0) \Rightarrow (\forall X1. (\neg v1\_xboole\_0 X1) \Rightarrow \\
& (\forall X2. ((v1\_funct\_1 X2) \wedge ((v1\_funct\_2 X2 (k2\_zfmisc\_1 X0 \\
& X0) X0) \wedge (m1\_subset\_1 X2 (k1\_zfmisc\_1 (k2\_zfmisc\_1 (k2\_zfmisc\_1 \\
& X0 X0) X0)))))) \Rightarrow (\forall X3. (m1\_subset\_1 X3 (k5\_finsub\_1 X1)) \Rightarrow \\
& (\forall X4. ((v1\_funct\_1 X4) \wedge ((v1\_funct\_2 X4 X1 X0) \wedge (m1\_subset\_1 \\
& X4 (k1\_zfmisc\_1 (k2\_zfmisc\_1 X1 X0)))))) \Rightarrow (((v3\_binop\_1 X2 X0) \wedge \\
& ((v1\_binop\_1 X2 X0) \wedge ((v2\_binop\_1 X2 X0) \wedge (v1\_setwiseo X2 X0)))) \Rightarrow \\
& (\forall X5. (m1\_subset\_1 X5 X1) \Rightarrow (k7\_setwiseo X1 X0 X2 (k5\_setwiseo \\
& X1 X3 (k2\_setwiseo X1 X5)) X4 = k5\_binop\_1 X0 X2 (k7\_setwiseo X1 X0 \\
& X2 X3 X4) (k3\_funct\_2 X1 X0 X4 X5))))))
\end{aligned} \tag{1}$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge(v6\_lattices X0)\wedge(l1\_lattices X0)))\wedge((m1\_subset\_1 X1 (u1\_struct\_0 X0))\wedge(m1\_subset\_1 X2 (u1\_struct\_0 X0))))\Rightarrow(k4\_lattices X0 X1 X2 = k2\_lattices X0 X1 X2) \quad (2)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0)\wedge(v7\_lattices X0)\wedge(l1\_lattices X0))\Rightarrow((v1\_funct\_1 (u1\_lattices X0))\wedge((v1\_funct\_2 (u1\_lattices X0) (k2\_zfmisc\_1 (u1\_struct\_0 X0) (u1\_struct\_0 X0)) (u1\_struct\_0 X0))\wedge(v2\_binop\_1 (u1\_lattices X0) (u1\_struct\_0 X0)))) \quad (3)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0)\wedge(v6\_lattices X0)\wedge(l1\_lattices X0))\Rightarrow((v1\_funct\_1 (u1\_lattices X0))\wedge((v1\_funct\_2 (u1\_lattices X0) (k2\_zfmisc\_1 (u1\_struct\_0 X0) (u1\_struct\_0 X0)) (u1\_struct\_0 X0))\wedge(v1\_binop\_1 (u1\_lattices X0) (u1\_struct\_0 X0)))) \quad (4)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0)\wedge(v10\_lattices X0)\wedge(l3\_lattices X0))\Rightarrow((v1\_funct\_1 (u1\_lattices X0))\wedge((v1\_funct\_2 (u1\_lattices X0) (k2\_zfmisc\_1 (u1\_struct\_0 X0) (u1\_struct\_0 X0)) (u1\_struct\_0 X0))\wedge(v3\_binop\_1 (u1\_lattices X0) (u1\_struct\_0 X0)))) \quad (5)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0)\wedge(l1\_struct\_0 X0))\Rightarrow(\neg v1\_xboole\_0 (u1\_struct\_0 X0)) \quad (6)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0)\wedge(v10\_lattices X0)\wedge((v14\_lattices X0)\wedge(l3\_lattices X0)))\Rightarrow((v1\_funct\_1 (u1\_lattices X0))\wedge((v1\_funct\_2 (u1\_lattices X0) (k2\_zfmisc\_1 (u1\_struct\_0 X0) (u1\_struct\_0 X0)) (u1\_struct\_0 X0))\wedge(v1\_setwiseo (u1\_lattices X0) (u1\_struct\_0 X0)))) \quad (7)$$

Assume the following.

$$\forall X0.(l1\_lattices X0)\Rightarrow((v1\_funct\_1 (u1\_lattices X0))\wedge((v1\_funct\_2 (u1\_lattices X0) (k2\_zfmisc\_1 (u1\_struct\_0 X0) (u1\_struct\_0 X0)) (u1\_struct\_0 X0))\wedge(m1\_subset\_1 (u1\_lattices X0) (k1\_zfmisc\_1 (k2\_zfmisc\_1 (k2\_zfmisc\_1 (u1\_struct\_0 X0) (u1\_struct\_0 X0)) (u1\_struct\_0 X0)))))) \quad (8)$$

Assume the following.

$$\forall X0.(l3\_lattices\ X0)\Rightarrow((l1\_lattices\ X0)\wedge(l2\_lattices\ X0)) \quad (9)$$

Assume the following.

$$\forall X0.(l1\_lattices\ X0)\Rightarrow(l1\_struct\_0\ X0) \quad (10)$$

Assume the following.

$$\begin{aligned} \forall X0.\forall X1.\forall X2.((m1\_subset\_1\ X1\ (k5\_finsub\_1 \\ X0))\wedge(m1\_subset\_1\ X2\ (k5\_finsub\_1\ X0)))\Rightarrow(m1\_subset\_1\ (k5\_setwiseo \\ X0\ X1\ X2)\ (k5\_finsub\_1\ X0)) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} \forall X0.\forall X1.\forall X2.\forall X3.((\neg v1\_xboole\_0\ X0)\wedge \\ (((\neg v2\_struct\_0\ X1)\wedge((v10\_lattices\ X1)\wedge(l3\_lattices\ X1)))\wedge \\ ((m1\_subset\_1\ X2\ (k5\_finsub\_1\ X0))\wedge((v1\_funct\_1\ X3)\wedge((v1\_funct\_2 \\ X3\ X0\ (u1\_struct\_0\ X1))\wedge(m1\_subset\_1\ X3\ (k1\_zfmisc\_1\ (k2\_zfmisc\_1 \\ X0\ (u1\_struct\_0\ X1))))))))\Rightarrow(m1\_subset\_1\ (k3\_lattice2\ X0\ X1\ X2 \\ X3)\ (u1\_struct\_0\ X1)) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} \forall X0.\forall X1.\forall X2.\forall X3.((\neg v1\_xboole\_0\ X0)\wedge \\ (((v1\_funct\_1\ X2)\wedge((v1\_funct\_2\ X2\ X0\ X1)\wedge(m1\_subset\_1\ X2\ (k1\_zfmisc\_1 \\ (k2\_zfmisc\_1\ X0\ X1)))))\wedge(m1\_subset\_1\ X3\ X0)))\Rightarrow(m1\_subset\_1\ ( \\ k3\_funct\_2\ X0\ X1\ X2\ X3)\ X1) \end{aligned} \quad (13)$$

Assume the following.

$$\forall X0.\forall X1.((\neg v1\_xboole\_0\ X0)\wedge(m1\_subset\_1\ X1\ X0))\Rightarrow \\ (m1\_subset\_1\ (k2\_setwiseo\ X0\ X1)\ (k5\_finsub\_1\ X0)) \quad (14)$$

Assume the following.

$$\begin{aligned} \forall X0.(\neg v1\_xboole\_0\ X0)\Rightarrow(\forall X1.((\neg v2\_struct\_0\ X1)\wedge \\ ((v10\_lattices\ X1)\wedge(l3\_lattices\ X1)))\Rightarrow(\forall X2.(m1\_subset\_1 \\ X2\ (k5\_finsub\_1\ X0))\Rightarrow(\forall X3.((v1\_funct\_1\ X3)\wedge((v1\_funct\_2 \\ X3\ X0\ (u1\_struct\_0\ X1))\wedge(m1\_subset\_1\ X3\ (k1\_zfmisc\_1\ (k2\_zfmisc\_1 \\ X0\ (u1\_struct\_0\ X1))))))\Rightarrow(k3\_lattice2\ X0\ X1\ X2\ X3 = k7\_setwiseo \\ X0\ (u1\_struct\_0\ X1)\ (u1\_lattices\ X1)\ X2\ X3))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} \forall X0.((\neg v2\_struct\_0\ X0)\wedge(l1\_lattices\ X0))\Rightarrow(\forall X1. \\ (m1\_subset\_1\ X1\ (u1\_struct\_0\ X0))\Rightarrow(\forall X2.(m1\_subset\_1\ X2 \\ (u1\_struct\_0\ X0))\Rightarrow(k2\_lattices\ X0\ X1\ X2 = k5\_binop\_1\ (u1\_struct\_0 \\ X0)\ (u1\_lattices\ X0)\ X1\ X2))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} \forall X0. (&l3\_lattices\ X0) \Rightarrow (((\neg v2\_struct\_0\ X0) \wedge (v10\_lattices \\ X0)) \Rightarrow &((\neg v2\_struct\_0\ X0) \wedge ((v4\_lattices\ X0) \wedge ((v5\_lattices\ X0) \wedge \\ ((v6\_lattices\ X0) \wedge &((v7\_lattices\ X0) \wedge ((v8\_lattices\ X0) \wedge (v9\_lattices \\ X0)))))))) & \end{aligned} \quad (17)$$

**Theorem 1**

$$\begin{aligned} \forall X0. (&(\neg v2\_struct\_0\ X0) \wedge ((v10\_lattices\ X0) \wedge ((v14\_lattices \\ X0) \wedge (&l3\_lattices\ X0)))) \Rightarrow (\forall X1. (m1\_subset\_1\ X1\ (k5\_finsub\_1 \\ (u1\_struct\_0\ X0))) \Rightarrow &(\forall X2. (m1\_subset\_1\ X2\ (u1\_struct\_0 \\ X0)) \Rightarrow (\forall X3. ((v1\_funct\_1\ X3) \wedge ((v1\_funct\_2\ X3\ (u1\_struct\_0 \\ X0)\ (u1\_struct\_0\ X0)) \wedge (m1\_subset\_1\ X3\ (k1\_zfmisc\_1\ (k2\_zfmisc\_1 \\ (u1\_struct\_0\ X0)\ (u1\_struct\_0\ X0)))))) \Rightarrow (k3\_lattice2\ (u1\_struct\_0 \\ X0)\ X0\ (k5\_setwiseo\ (u1\_struct\_0\ X0)\ X1\ (k2\_setwiseo\ (u1\_struct\_0 \\ X0)\ X2))\ X3 = k4\_lattices\ X0\ (k3\_lattice2\ (u1\_struct\_0\ X0)\ X0\ X1\ X3) \\ (k3\_funct\_2\ (u1\_struct\_0\ X0)\ (u1\_struct\_0\ X0)\ X3\ X2)))))) \end{aligned}$$