

## t22\_closure2

(TMFnqx5t3oeZeSi11BNbHym5dDDaunrVPK7)

October 27, 2020

Let  $v1\_relat\_1 : \iota \Rightarrow o$  be given. Let  $v4\_relat\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v1\_funct\_1 : \iota \Rightarrow o$  be given. Let  $v1\_partfun1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k1\_zfmisc\_1 : \iota \Rightarrow \iota$  be given. Let  $k1\_closure2 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k2\_tarski : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $r6\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k2\_mboolean : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k5\_closure2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k2\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k3\_tarski : \iota \Rightarrow \iota$  be given. Let  $k2\_xboole\_0 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_funct\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k4\_closure2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall X0. \forall X1. k3\_tarski (k2\_tarski X0 X1) = k2\_xboole\_0 X0 X1 \quad (1)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. ((v1\_relat\_1 X2) \wedge ((v4\_relat\_1 \\ & X2 X1) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X1)))) \Rightarrow (\forall X3. ( \\ & (v1\_relat\_1 X3) \wedge (v1\_funct\_1 X3)) \Rightarrow (\forall X4. ((v1\_relat\_1 X4) \wedge \\ & (v1\_funct\_1 X4)) \Rightarrow (\forall X5. (m1\_subset\_1 X5 (k1\_zfmisc\_1 (k1\_closure2 \\ & X1 X2)))) \Rightarrow (((X0 \in X1) \wedge (X5 = k2\_tarski X3 X4)) \Rightarrow (k1\_funct\_1 (k4\_closure2 \\ & X1 X2 X5) X0 = k2\_tarski (k1\_funct\_1 X3 X0) (k1\_funct\_1 X4 X0)))))) \end{aligned} \quad (2)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 \\ & X1 X0) \wedge ((v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0)))) \wedge ((v1\_relat\_1 \\ & X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))))) \Rightarrow \\ & (r6\_pboole X0 X1 X2) \Leftrightarrow (X1 = X2) \end{aligned} \quad (3)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 \\ & X1 X0) \wedge ((v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0)))) \wedge (m1\_subset\_1 \\ & X2 (k1\_zfmisc\_1 (k1\_closure2 X0 X1)))) \Rightarrow (k5\_closure2 X0 X1 X2 = k4\_closure2 \\ & X0 X1 X2) \end{aligned} \quad (4)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 \\ & X1 X0) \wedge ((v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0)))) \wedge (m1\_subset\_1 \\ & X2 (k1\_zfmisc\_1 (k1\_closure2 X0 X1)))) \Rightarrow ((v1\_relat\_1 (k4\_closure2 \\ & X0 X1 X2)) \wedge ((v4\_relat\_1 (k4\_closure2 X0 X1 X2) X0) \wedge ((v1\_funct\_1 \\ & (k4\_closure2 X0 X1 X2)) \wedge (v1\_partfun1 (k4\_closure2 X0 X1 X2) X0)))) \end{aligned} \quad (5)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge ( \\ & (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0)))) \Rightarrow ((v1\_relat\_1 (k2\_mboolean \\ & X0 X1)) \wedge ((v4\_relat\_1 (k2\_mboolean X0 X1) X0) \wedge ((v1\_funct\_1 (k2\_mboolean \\ & X0 X1)) \wedge (v1\_partfun1 (k2\_mboolean X0 X1) X0)))) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge ( \\ & (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0)))) \Rightarrow (\forall X2. ((v1\_relat\_1 \\ & X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\ & (\forall X3. ((v1\_relat\_1 X3) \wedge ((v4\_relat\_1 X3 X0) \wedge ((v1\_funct\_1 \\ & X3) \wedge (v1\_partfun1 X3 X0)))) \Rightarrow ((X3 = k2\_pboole X0 X1 X2) \Leftrightarrow (\forall X4. \\ & (X4 \in X0) \Rightarrow (k1\_funct\_1 X3 X4 = k2\_xboole\_0 (k1\_funct\_1 X1 X4) (k1\_funct\_1 \\ & X2 X4)))))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge ( \\ & (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0)))) \Rightarrow (\forall X2. ((v1\_relat\_1 \\ & X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\ & ((X2 = k2\_mboolean X0 X1) \Leftrightarrow (\forall X3. (X3 \in X0) \Rightarrow (k1\_funct\_1 X2 X3 = \\ & k3\_tarski (k1\_funct\_1 X1 X3)))))) \end{aligned} \quad (8)$$

**Theorem 1**

$$\begin{aligned} & \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge ( \\ & (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0)))) \Rightarrow (\forall X2. ((v1\_relat\_1 \\ & X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\ & (\forall X3. ((v1\_relat\_1 X3) \wedge ((v4\_relat\_1 X3 X0) \wedge ((v1\_funct\_1 \\ & X3) \wedge (v1\_partfun1 X3 X0)))) \Rightarrow (\forall X4. (m1\_subset\_1 X4 (k1\_zfmisc\_1 \\ & (k1\_closure2 X0 X1)) \Rightarrow ((X4 = k2\_tarski X2 X3) \Rightarrow (r6\_pboole X0 (k2\_mboolean \\ & X0 (k5\_closure2 X0 X1 X4) (k2\_pboole X0 X2 X3)))))) \end{aligned}$$