

## t22\_lattice3

(TMWLfLFJ6nrFgvXvDn1DCWe5PsW7WjhU4bo)

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Let  $v2\_struct\_0 : \iota \Rightarrow o$  be given. Let  $v10\_lattices : \iota \Rightarrow o$  be given. Let  $l3\_lattices : \iota \Rightarrow o$  be given. Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $u1\_struct\_0 : \iota \Rightarrow \iota$  be given. Let  $r4\_lattice3 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k7\_domain\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $r3\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k3\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v5\_lattices : \iota \Rightarrow o$  be given. Let  $l2\_lattices : \iota \Rightarrow o$  be given. Let  $r1\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v4\_lattices : \iota \Rightarrow o$  be given. Let  $v6\_lattices : \iota \Rightarrow o$  be given. Let  $v8\_lattices : \iota \Rightarrow o$  be given. Let  $v9\_lattices : \iota \Rightarrow o$  be given. Let  $k1\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v1\_xboole\_0 : \iota \Rightarrow o$  be given. Let  $k2\_tarski : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $l1\_struct\_0 : \iota \Rightarrow o$  be given. Let  $l1\_lattices : \iota \Rightarrow o$  be given. Let  $v7\_lattices : \iota \Rightarrow o$  be given. Assume the following.

$$\begin{aligned} & \forall X0. ((\neg v2\_struct\_0 X0) \wedge ((v5\_lattices X0) \wedge (l2\_lattices \\ & X0))) \Rightarrow (\forall X1. (m1\_subset\_1 X1 (u1\_struct\_0 X0)) \Rightarrow (\forall X2. \\ & (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (\forall X3. (m1\_subset\_1 X3 \\ & (u1\_struct\_0 X0)) \Rightarrow (((r1\_lattices X0 X1 X2) \wedge (r1\_lattices X0 X2 \\ & X3)) \Rightarrow (r1\_lattices X0 X1 X3)))))) \end{aligned} \quad (1)$$

Assume the following.

$$\begin{aligned} & \forall X0. ((\neg v2\_struct\_0 X0) \wedge ((v4\_lattices X0) \wedge ((v5\_lattices \\ & X0) \wedge ((v6\_lattices X0) \wedge ((v8\_lattices X0) \wedge ((v9\_lattices X0) \wedge \\ & (l3\_lattices X0)))))))) \Rightarrow (\forall X1. (m1\_subset\_1 X1 (u1\_struct\_0 \\ & X0)) \Rightarrow (\forall X2. (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (\forall X3. \\ & (m1\_subset\_1 X3 (u1\_struct\_0 X0)) \Rightarrow (((r3\_lattices X0 X1 X3) \wedge (r3\_lattices \\ & X0 X2 X3)) \Rightarrow (r3\_lattices X0 (k3\_lattices X0 X1 X2) X3)))))) \end{aligned} \quad (2)$$

Assume the following.

$$\begin{aligned} & \forall X0. ((\neg v2\_struct\_0 X0) \wedge ((v5\_lattices X0) \wedge ((v6\_lattices \\ & X0) \wedge ((v8\_lattices X0) \wedge ((v9\_lattices X0) \wedge (l3\_lattices X0)))))) \Rightarrow \\ & (\forall X1. (m1\_subset\_1 X1 (u1\_struct\_0 X0)) \Rightarrow (\forall X2. (m1\_subset\_1 \\ & X2 (u1\_struct\_0 X0)) \Rightarrow (r1\_lattices X0 X1 (k1\_lattices X0 X1 X2)))) \end{aligned} \quad (3)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge((v6\_lattices \\ & X0)\wedge((v8\_lattices X0)\wedge((v9\_lattices X0)\wedge(l3\_lattices X0))))))\wedge \\ & ((m1\_subset\_1 X1 (u1\_struct\_0 X0))\wedge(m1\_subset\_1 X2 (u1\_struct\_0 \\ & X0)))\Rightarrow((r3\_lattices X0 X1 X2)\Leftrightarrow(r1\_lattices X0 X1 X2)) \end{aligned} \quad (4)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.((\neg v1\_xboole\_0 X0)\wedge((m1\_subset\_1 \\ & X1 X0)\wedge(m1\_subset\_1 X2 X0)))\Rightarrow(k7\_domain\_1 X0 X1 X2 = k2\_tarski X1 \\ & X2) \end{aligned} \quad (5)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge((v4\_lattices \\ & X0)\wedge(l2\_lattices X0)))\wedge((m1\_subset\_1 X1 (u1\_struct\_0 X0))\wedge( \\ & m1\_subset\_1 X2 (u1\_struct\_0 X0))))\Rightarrow(k3\_lattices X0 X1 X2 = k1\_lattices \\ & X0 X1 X2) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} & \forall X0.((\neg v2\_struct\_0 X0)\wedge(l1\_struct\_0 X0))\Rightarrow(\neg v1\_xboole\_0 \\ & (u1\_struct\_0 X0)) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} & \forall X0.(l3\_lattices X0)\Rightarrow((l1\_lattices X0)\wedge(l2\_lattices X0)) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & \forall X0.(l2\_lattices X0)\Rightarrow(l1\_struct\_0 X0) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge((v4\_lattices \\ & X0)\wedge(l2\_lattices X0)))\wedge((m1\_subset\_1 X1 (u1\_struct\_0 X0))\wedge( \\ & m1\_subset\_1 X2 (u1\_struct\_0 X0))))\Rightarrow(m1\_subset\_1 (k3\_lattices \\ & X0 X1 X2) (u1\_struct\_0 X0)) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(X2 = k2\_tarski X0 X1)\Leftrightarrow(\forall X3. \\ & (X3 \in X2)\Leftrightarrow((X3 = X0)\vee(X3 = X1))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} & \forall X0.((\neg v2\_struct\_0 X0)\wedge(l3\_lattices X0))\Rightarrow(\forall X1. \\ & (m1\_subset\_1 X1 (u1\_struct\_0 X0))\Rightarrow(\forall X2.(r4\_lattice3 X0 \\ & X1 X2)\Leftrightarrow(\forall X3.(m1\_subset\_1 X3 (u1\_struct\_0 X0))\Rightarrow((X3 \in X2)\Rightarrow \\ & (r1\_lattices X0 X3 X1)))))) \end{aligned} \quad (12)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge(v4\_lattices X0)\wedge(l2\_lattices X0))\wedge((m1\_subset\_1 X1 (u1\_struct\_0 X0))\wedge(m1\_subset\_1 X2 (u1\_struct\_0 X0))))\Rightarrow(k3\_lattices X0 X1 X2 = k3\_lattices X0 X2 X1) \quad (13)$$

Assume the following.

$$\forall X0.\forall X1.k2\_tarSKI X0 X1 = k2\_tarSKI X1 X0 \quad (14)$$

Assume the following.

$$\forall X0.(l3\_lattices X0)\Rightarrow(((\neg v2\_struct\_0 X0)\wedge(v10\_lattices X0))\Rightarrow((\neg v2\_struct\_0 X0)\wedge(v4\_lattices X0)\wedge((v5\_lattices X0)\wedge((v6\_lattices X0)\wedge((v7\_lattices X0)\wedge((v8\_lattices X0)\wedge(v9\_lattices X0)))))))) \quad (15)$$

**Theorem 1**

$$\forall X0.((\neg v2\_struct\_0 X0)\wedge((v10\_lattices X0)\wedge(l3\_lattices X0)))\Rightarrow(\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 X0))\Rightarrow(\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 X0))\Rightarrow(\forall X3.(m1\_subset\_1 X3 (u1\_struct\_0 X0))\Rightarrow((r4\_lattice3 X0 X1 (k7\_domain\_1 (u1\_struct\_0 X0) X2 X3))\Leftrightarrow(r3\_lattices X0 (k3\_lattices X0 X2 X3) X1))))))$$