

## t24\_sublemma

(TMUXN8xW6sYZtyFctoqapVcS8QKAaYku9xC)

October 27, 2020

Let  $m1\_qc\_lang1 : \iota \Rightarrow o$  be given. Let  $v1\_xboole\_0 : \iota \Rightarrow o$  be given. Let  $m1\_valuat\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $m2\_funct\_2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k3\_qc\_lang1 : \iota \Rightarrow \iota$  be given. Let  $k2\_valuat\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $m2\_subset\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k16\_subst1 : \iota \Rightarrow \iota$  be given. Let  $k38\_subst1 : \iota \Rightarrow \iota$  be given. Let  $k19\_subst1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $r1\_sublemma : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k1\_sublemma : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k3\_sublemma : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k6\_sublemma : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k2\_sublemma : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k7\_cqc\_lang : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k9\_qc\_lang1 : \iota \Rightarrow \iota$  be given. Let  $k3\_cqc\_lang : \iota \Rightarrow \iota$  be given. Let  $r1\_valuat\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k1\_zfmisc\_1 : \iota \Rightarrow \iota$  be given. Let  $m1\_funct\_2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k1\_xtuple\_0 : \iota \Rightarrow \iota$  be given. Let  $k2\_xtuple\_0 : \iota \Rightarrow \iota$  be given. Let  $v1\_funct\_1 : \iota \Rightarrow o$  be given. Let  $k2\_zfmisc\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k4\_relset\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k2\_subst1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall X0.(m1\_qc\_lang1 X0) \Rightarrow (\forall X1.(m2\_subset\_1 X1 (k16\_subst1 \\ & X0) (k38\_subst1 X0)) \Rightarrow (\forall X2.(m2\_subset\_1 X2 (k16\_subst1 \\ & X0) (k38\_subst1 X0)) \Rightarrow ((k19\_subst1 X0 X1 = k19\_subst1 X0 X2) \Rightarrow \\ & ((k2\_sublemma X0 (k6\_sublemma X0 X1 X2) = k7\_cqc\_lang X0 (k2\_sublemma \\ & X0 X1) (k2\_sublemma X0 X2)) \wedge (k19\_subst1 X0 (k6\_sublemma X0 X1 \\ & X2) = k19\_subst1 X0 X1)))))) \end{aligned} \tag{1}$$

Assume the following.

$$\begin{aligned} & \forall X0.(m1\_qc\_lang1 X0) \Rightarrow (\forall X1.(\neg v1\_xboole\_0 X1) \Rightarrow ( \\ & \forall X2.(m2\_funct\_2 X2 (k3\_qc\_lang1 X0) X1 (k2\_valuat\_1 X0 X1)) \Rightarrow \\ & (\forall X3.(m2\_subset\_1 X3 (k9\_qc\_lang1 X0) (k3\_cqc\_lang X0)) \Rightarrow \\ & (\forall X4.(m2\_subset\_1 X4 (k9\_qc\_lang1 X0) (k3\_cqc\_lang X0)) \Rightarrow \\ & (\forall X5.(m1\_valuat\_1 X5 X0 X1) \Rightarrow ((r1\_valuat\_1 X0 X1 (k7\_cqc\_lang \\ & X0 X3 X4) X5 X2) \Leftrightarrow ((r1\_valuat\_1 X0 X1 X3 X5 X2) \wedge (r1\_valuat\_1 X0 X1 X4 \\ & X5 X2))))))))) \end{aligned} \tag{2}$$

Assume the following.

$$\forall X0.\forall X1.((\neg v1\_xboole\_0 X0)\wedge((\neg v1\_xboole\_0 X1)\wedge(m1\_subset\_1 X1 (k1\_zfmisc\_1 X0))))\Rightarrow(\forall X2.(m2\_subset\_1 X2 X0 X1)\Leftrightarrow(m1\_subset\_1 X2 X1)) \quad (3)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.((\neg v1\_xboole\_0 X1)\wedge(m1\_funct\_2 X2 X0 X1))\Rightarrow(\forall X3.(m2\_funct\_2 X3 X0 X1 X2)\Leftrightarrow(m1\_subset\_1 X3 X2)) \quad (4)$$

Assume the following.

$$\forall X0.\forall X1.((m1\_qc\_lang1 X0)\wedge(m1\_subset\_1 X1 (k38\_substitut1 X0)))\Rightarrow(k2\_sublemma X0 X1 = k1\_xtuple\_0 X1) \quad (5)$$

Assume the following.

$$\forall X0.\forall X1.((m1\_qc\_lang1 X0)\wedge(m1\_subset\_1 X1 (k16\_substitut1 X0)))\Rightarrow(k19\_substitut1 X0 X1 = k2\_xtuple\_0 X1) \quad (6)$$

Assume the following.

$$\forall X0.(m1\_qc\_lang1 X0)\Rightarrow(\neg v1\_xboole\_0 (k38\_substitut1 X0)) \quad (7)$$

Assume the following.

$$\forall X0.\forall X1.((\neg v1\_xboole\_0 X0)\wedge((\neg v1\_xboole\_0 X1)\wedge(m1\_subset\_1 X1 (k1\_zfmisc\_1 X0))))\Rightarrow(\forall X2.(m2\_subset\_1 X2 X0 X1)\Rightarrow(m1\_subset\_1 X2 X0)) \quad (8)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.((m1\_qc\_lang1 X0)\wedge((m1\_subset\_1 X1 (k38\_substitut1 X0))\wedge(m1\_subset\_1 X2 (k38\_substitut1 X0))))\Rightarrow(m2\_subset\_1 (k6\_sublemma X0 X1 X2) (k16\_substitut1 X0) (k38\_substitut1 X0)) \quad (9)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.\forall X3.((m1\_qc\_lang1 X0)\wedge((m1\_subset\_1 X1 (k38\_substitut1 X0))\wedge((\neg v1\_xboole\_0 X2)\wedge(m1\_subset\_1 X3 (k2\_valuat\_1 X0 X2))))\Rightarrow((v1\_funct\_1 (k3\_sublemma X0 X1 X2 X3))\wedge(m1\_subset\_1 (k3\_sublemma X0 X1 X2 X3) (k1\_zfmisc\_1 (k2\_zfmisc\_1 (k3\_qc\_lang1 X0) X2)))) \quad (10)$$

Assume the following.

$$\forall X0.(m1\_qc\_lang1 X0)\Rightarrow(m1\_subset\_1 (k38\_substitut1 X0) (k1\_zfmisc\_1 (k16\_substitut1 X0))) \quad (11)$$

Assume the following.

$$\forall X0.\forall X1.((m1\_qc\_lang1\ X0)\wedge(\neg v1\_xboole\_0\ X1))\Rightarrow (m1\_funct\_2\ (k2\_valuat\_1\ X0\ X1)\ (k3\_qc\_lang1\ X0\ X1)) \quad (12)$$

Assume the following.

$$\forall X0.\forall X1.((m1\_qc\_lang1\ X0)\wedge(m1\_subset\_1\ X1\ (k38\_subst1\ X0)))\Rightarrow(m2\_subset\_1\ (k2\_sublemma\ X0\ X1)\ (k9\_qc\_lang1\ X0)\ (k3\_cqc\_lang\ X0)) \quad (13)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.\forall X3.((m1\_qc\_lang1\ X0)\wedge((\neg v1\_xboole\_0\ X1)\wedge((m1\_subset\_1\ X2\ (k2\_valuat\_1\ X0\ X1))\wedge((v1\_funct\_1\ X3)\wedge(m1\_subset\_1\ X3\ (k1\_zfmisc\_1\ (k2\_zfmisc\_1\ (k3\_qc\_lang1\ X0\ X1))))))))\Rightarrow(m2\_funct\_2\ (k1\_sublemma\ X0\ X1\ X2\ X3)\ (k3\_qc\_lang1\ X0\ X1)\ (k2\_valuat\_1\ X0\ X1)) \quad (14)$$

Assume the following.

$$\forall X0.(m1\_qc\_lang1\ X0)\Rightarrow(\forall X1.(m2\_subset\_1\ X1\ (k16\_subst1\ X0)\ (k38\_subst1\ X0))\Rightarrow(\forall X2.(\neg v1\_xboole\_0\ X2)\Rightarrow(\forall X3.(m2\_funct\_2\ X3\ (k3\_qc\_lang1\ X0)\ X2\ (k2\_valuat\_1\ X0\ X2))\Rightarrow(\forall X4.(m1\_valuat\_1\ X4\ X0\ X2)\Rightarrow((r1\_sublemma\ X0\ X1\ X2\ X3\ X4)\Leftrightarrow(r1\_valuat\_1\ X0\ X2\ (k2\_sublemma\ X0\ X1)\ X4\ X3)))))) \quad (15)$$

Assume the following.

$$\forall X0.(m1\_qc\_lang1\ X0)\Rightarrow(\forall X1.(m2\_subset\_1\ X1\ (k16\_subst1\ X0)\ (k38\_subst1\ X0))\Rightarrow(\forall X2.(\neg v1\_xboole\_0\ X2)\Rightarrow(\forall X3.(m2\_funct\_2\ X3\ (k3\_qc\_lang1\ X0)\ X2\ (k2\_valuat\_1\ X0\ X2))\Rightarrow(k3\_sublemma\ X0\ X1\ X2\ X3 = k4\_relset\_1\ (k3\_qc\_lang1\ X0)\ (k3\_qc\_lang1\ X0)\ (k3\_qc\_lang1\ X0)\ X2\ (k2\_subst1\ X0\ (k19\_subst1\ X0\ X1))\ X3)))) \quad (16)$$

Assume the following.

$$\forall X0.(v1\_xboole\_0\ X0)\Rightarrow(\forall X1.(m1\_subset\_1\ X1\ (k1\_zfmisc\_1\ X0))\Rightarrow(v1\_xboole\_0\ X1)) \quad (17)$$

**Theorem 1**

$$\begin{aligned} & \forall X0.(m1\_qc\_lang1\ X0) \Rightarrow (\forall X1.(\neg v1\_xboole\_0\ X1) \Rightarrow ( \\ & \quad \forall X2.(m1\_valuat\_1\ X2\ X0\ X1) \Rightarrow (\forall X3.(m2\_funct\_2\ X3\ ( \\ & \quad k3\_qc\_lang1\ X0)\ X1\ (k2\_valuat\_1\ X0\ X1)) \Rightarrow (\forall X4.(m2\_subset\_1 \\ & \quad X4\ (k16\_subst1\ X0)\ (k38\_subst1\ X0)) \Rightarrow (\forall X5.(m2\_subset\_1 \\ & \quad X5\ (k16\_subst1\ X0)\ (k38\_subst1\ X0)) \Rightarrow ((k19\_subst1\ X0\ X4 = \\ & \quad k19\_subst1\ X0\ X5) \Rightarrow (((r1\_sublemma\ X0\ X4\ X1\ (k1\_sublemma\ X0\ X1\ X3 \\ & \quad (k3\_sublemma\ X0\ X4\ X1\ X3))\ X2) \wedge (r1\_sublemma\ X0\ X5\ X1\ (k1\_sublemma \\ & \quad X0\ X1\ X3\ (k3\_sublemma\ X0\ X5\ X1\ X3))\ X2)) \Leftrightarrow (r1\_sublemma\ X0\ (k6\_sublemma \\ & \quad X0\ X4\ X5)\ X1\ (k1\_sublemma\ X0\ X1\ X3\ (k3\_sublemma\ X0\ (k6\_sublemma\ X0 \\ & \quad X4\ X5)\ X1\ X3))\ X2))))))))) \end{aligned}$$