

# t25\_monoid\_1

(TMb5PP8wtMC1U6MCDAf5WtTvQobt51kDyns)

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Let  $v2\_struct\_0 : \iota \Rightarrow o$  be given. Let  $v1\_group\_1 : \iota \Rightarrow o$  be given. Let  $l3\_algstr\_0 : \iota \Rightarrow o$  be given. Let  $m2\_monoid\_0 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k4\_binop\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $u1\_struct\_0 : \iota \Rightarrow \iota$  be given. Let  $u2\_algstr\_0 : \iota \Rightarrow \iota$  be given. Let  $m3\_monoid\_0 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k9\_monoid\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k11\_monoid\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v1\_xboole\_0 : \iota \Rightarrow o$  be given. Let  $k5\_struct\_0 : \iota \Rightarrow \iota$  be given. Let  $k5\_monoid\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k9\_funct\_2 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $r1\_funct\_2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k2\_zfmisc\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k8\_monoid\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v1\_funct\_1 : \iota \Rightarrow o$  be given. Let  $v1\_funct\_2 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k1\_zfmisc\_1 : \iota \Rightarrow \iota$  be given. Let  $r1\_relset\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $r1\_tarski : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k1\_funct\_2 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k2\_funcop\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $l1\_struct\_0 : \iota \Rightarrow o$  be given. Let  $v22\_algstr\_0 : \iota \Rightarrow o$  be given. Let  $v4\_vectsp\_1 : \iota \Rightarrow o$  be given. Let  $v1\_monoid\_0 : \iota \Rightarrow o$  be given. Let  $l4\_algstr\_0 : \iota \Rightarrow o$  be given. Assume the following.

$$\forall X0. \forall X1. \neg (X0 \in X1) \wedge (v1\_xboole\_0 X1) \tag{1}$$

Assume the following.

$$\begin{aligned} & \forall X0. ((\neg v2\_struct\_0 X0) \wedge (l3\_algstr\_0 X0)) \Rightarrow (\forall X1. \\ & ((\neg v2\_struct\_0 X1) \wedge (m2\_monoid\_0 X1 X0)) \Rightarrow (((v1\_group\_1 X0) \wedge \\ & k4\_binop\_1 (u1\_struct\_0 X0) (u2\_algstr\_0 X0) \in u1\_struct\_0 X1)) \Rightarrow \\ & ((v1\_group\_1 X1) \wedge (k4\_binop\_1 (u1\_struct\_0 X0) (u2\_algstr\_0 X0) = \\ & k4\_binop\_1 (u1\_struct\_0 X1) (u2\_algstr\_0 X1)))) \end{aligned} \tag{2}$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((\neg v2\_struct\_0 X1) \wedge ((v1\_group\_1 X1) \wedge \\ & l3\_algstr\_0 X1)) \Rightarrow (k5\_struct\_0 (k11\_monoid\_1 X1 X0) = k5\_monoid\_1 \\ & (u1\_struct\_0 X1) X0 (k4\_binop\_1 (u1\_struct\_0 X1) (u2\_algstr\_0 \\ & X1))) \end{aligned} \tag{3}$$

Assume the following.

$$\forall X0.\forall X1.(X0 \in X1) \Rightarrow (m1\_subset\_1 X0 X1) \quad (4)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.((\neg v2\_struct\_0 X1) \wedge (l3\_algstr\_0 X1)) \Rightarrow \\ & ((u1\_struct\_0 (k9\_monoid\_1 X1 X0) = k9\_funct\_2 X0 (u1\_struct\_0 \\ & X1)) \wedge (r1\_funct\_2 (k2\_zfmisc\_1 (u1\_struct\_0 (k9\_monoid\_1 X1 X0)) \\ & (u1\_struct\_0 (k9\_monoid\_1 X1 X0))) (u1\_struct\_0 (k9\_monoid\_1 \\ & X1 X0)) (k2\_zfmisc\_1 (k9\_funct\_2 X0 (u1\_struct\_0 X1)) (k9\_funct\_2 \\ & X0 (u1\_struct\_0 X1))) (k9\_funct\_2 X0 (u1\_struct\_0 X1)) (u2\_algstr\_0 \\ & (k9\_monoid\_1 X1 X0)) (k8\_monoid\_1 (u1\_struct\_0 X1) (u1\_struct\_0 \\ & X1) (u1\_struct\_0 X1) (u2\_algstr\_0 X1) X0))) \end{aligned} \quad (5)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.(\neg v1\_xboole\_0 X1) \Rightarrow (\forall X2.(\neg v1\_xboole\_0 \\ & X2) \Rightarrow (\forall X3.(\neg v1\_xboole\_0 X3) \Rightarrow (\forall X4.(\neg v1\_xboole\_0 \\ & X4) \Rightarrow (\forall X5.(\neg v1\_xboole\_0 X5) \Rightarrow (\forall X6.(\neg v1\_xboole\_0 \\ & X6) \Rightarrow (\forall X7.((v1\_funct\_1 X7) \wedge ((v1\_funct\_2 X7 (k2\_zfmisc\_1 \\ & X1 X2) X3) \wedge (m1\_subset\_1 X7 (k1\_zfmisc\_1 (k2\_zfmisc\_1 (k2\_zfmisc\_1 \\ & X1 X2) X3)))) \Rightarrow (\forall X8.((v1\_funct\_1 X8) \wedge ((v1\_funct\_2 X8 ( \\ & k2\_zfmisc\_1 X4 X5) X6) \wedge (m1\_subset\_1 X8 (k1\_zfmisc\_1 (k2\_zfmisc\_1 \\ & (k2\_zfmisc\_1 X4 X5) X6)))) \Rightarrow ((r1\_relset\_1 (k2\_zfmisc\_1 X1 X2) \\ & X3 X7 X8) \Rightarrow (r1\_relset\_1 (k2\_zfmisc\_1 (k9\_funct\_2 X0 X1) (k9\_funct\_2 \\ & X0 X2)) (k9\_funct\_2 X0 X3) (k8\_monoid\_1 X1 X2 X3 X7 X0) (k8\_monoid\_1 \\ & X4 X5 X6 X8 X0)))))))))) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.\forall X3.(m1\_subset\_1 X2 ( \\ & k1\_zfmisc\_1 (k2\_zfmisc\_1 X0 X1))) \Rightarrow ((r1\_relset\_1 X0 X1 X2 X3) \Leftrightarrow ( \\ & r1\_tarski X2 X3)) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.\forall X3.\forall X4.\forall X5. \\ & ((\neg v1\_xboole\_0 X1) \wedge ((\neg v1\_xboole\_0 X3) \wedge (((v1\_funct\_1 X4) \wedge (( \\ & v1\_funct\_2 X4 X0 X1) \wedge (m1\_subset\_1 X4 (k1\_zfmisc\_1 (k2\_zfmisc\_1 \\ & X0 X1)))))) \wedge ((v1\_funct\_1 X5) \wedge ((v1\_funct\_2 X5 X2 X3) \wedge (m1\_subset\_1 \\ & X5 (k1\_zfmisc\_1 (k2\_zfmisc\_1 X2 X3)))))) \Rightarrow ((r1\_funct\_2 X0 X1 \\ & X2 X3 X4 X5) \Leftrightarrow (X4 = X5)) \end{aligned} \quad (8)$$

Assume the following.

$$\forall X0.\forall X1.(\neg v1\_xboole\_0 X1) \Rightarrow (k9\_funct\_2 X0 X1 = k1\_funct\_2 X0 X1) \quad (9)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.((\neg v1\_xboole\_0 X0)\wedge(m1\_subset\_1 X2 X0))\Rightarrow(k5\_monoid\_1 X0 X1 X2 = k2\_funcop\_1 X1 X2) \quad (10)$$

Assume the following.

$$\forall X0.\forall X1.((\neg v2\_struct\_0 X0)\wedge((v1\_group\_1 X0)\wedge(l3\_algstr\_0 X0)))\Rightarrow(k11\_monoid\_1 X0 X1 = k9\_monoid\_1 X0 X1) \quad (11)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0)\wedge(l1\_struct\_0 X0))\Rightarrow(\neg v1\_xboole\_0 (u1\_struct\_0 X0)) \quad (12)$$

Assume the following.

$$\forall X0.\forall X1.(\neg v1\_xboole\_0 X1)\Rightarrow(\neg v1\_xboole\_0 (k1\_funct\_2 X0 X1)) \quad (13)$$

Assume the following.

$$\forall X0.(l3\_algstr\_0 X0)\Rightarrow((v1\_funct\_1 (u2\_algstr\_0 X0))\wedge((v1\_funct\_2 (u2\_algstr\_0 X0) (k2\_zfmisc\_1 (u1\_struct\_0 X0) (u1\_struct\_0 X0)) (u1\_struct\_0 X0))\wedge(m1\_subset\_1 (u2\_algstr\_0 X0) (k1\_zfmisc\_1 (k2\_zfmisc\_1 (k2\_zfmisc\_1 (u1\_struct\_0 X0) (u1\_struct\_0 X0)) (u1\_struct\_0 X0)))))) \quad (14)$$

Assume the following.

$$\forall X0.(l3\_algstr\_0 X0)\Rightarrow(\forall X1.(m2\_monoid\_0 X1 X0)\Rightarrow(l3\_algstr\_0 X1)) \quad (15)$$

Assume the following.

$$\forall X0.(l3\_algstr\_0 X0)\Rightarrow(l1\_struct\_0 X0) \quad (16)$$

Assume the following.

$$\forall X0.\forall X1.((\neg v2\_struct\_0 X0)\wedge(l3\_algstr\_0 X0))\Rightarrow(l3\_algstr\_0 (k9\_monoid\_1 X0 X1)) \quad (17)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.\forall X3.\forall X4.((\neg v1\_xboole\_0 X0)\wedge((\neg v1\_xboole\_0 X1)\wedge((\neg v1\_xboole\_0 X2)\wedge((v1\_funct\_1 X3)\wedge((v1\_funct\_2 X3 (k2\_zfmisc\_1 X0 X1) X2)\wedge(m1\_subset\_1 X3 (k1\_zfmisc\_1 (k2\_zfmisc\_1 (k2\_zfmisc\_1 X0 X1) X2))))))))))\Rightarrow((v1\_funct\_1 (k8\_monoid\_1 X0 X1 X2 X3 X4))\wedge((v1\_funct\_2 (k8\_monoid\_1 X0 X1 X2 X3 X4) (k2\_zfmisc\_1 (k9\_funct\_2 X4 X0) (k9\_funct\_2 X4 X1)) (k9\_funct\_2 X4 X2))\wedge(m1\_subset\_1 (k8\_monoid\_1 X0 X1 X2 X3 X4) (k1\_zfmisc\_1 (k2\_zfmisc\_1 (k2\_zfmisc\_1 (k9\_funct\_2 X4 X0) (k9\_funct\_2 X4 X1)) (k9\_funct\_2 X4 X2)))))) \quad (18)$$

Assume the following.

$$\forall X0.\forall X1.((v1\_funct\_1 X1)\wedge((v1\_funct\_2 X1 (k2\_zfmisc\_1 X0 X0) X0)\wedge(m1\_subset\_1 X1 (k1\_zfmisc\_1 (k2\_zfmisc\_1 (k2\_zfmisc\_1 X0 X0) X0))))\Rightarrow(m1\_subset\_1 (k4\_binop\_1 X0 X1) X0)) \quad (19)$$

Assume the following.

$$\forall X0.\forall X1.((\neg v2\_struct\_0 X0)\wedge((v1\_group\_1 X0)\wedge(l3\_algstr\_0 X0))\Rightarrow((\neg v2\_struct\_0 (k11\_monoid\_1 X0 X1))\wedge((v22\_algstr\_0 (k11\_monoid\_1 X0 X1))\wedge((v4\_vectsp\_1 (k11\_monoid\_1 X0 X1))\wedge((v1\_monoid\_0 (k11\_monoid\_1 X0 X1))\wedge(l4\_algstr\_0 (k11\_monoid\_1 X0 X1))))))) \quad (20)$$

Assume the following.

$$\forall X0.(l4\_algstr\_0 X0)\Rightarrow(\forall X1.(l4\_algstr\_0 X1)\Rightarrow((m3\_monoid\_0 X1 X0)\Leftrightarrow((r1\_tarski (u2\_algstr\_0 X1) (u2\_algstr\_0 X0))\wedge(k5\_struct\_0 X1 = k5\_struct\_0 X0)))) \quad (21)$$

Assume the following.

$$\forall X0.(l3\_algstr\_0 X0)\Rightarrow(\forall X1.(l3\_algstr\_0 X1)\Rightarrow((m2\_monoid\_0 X1 X0)\Leftrightarrow(r1\_tarski (u2\_algstr\_0 X1) (u2\_algstr\_0 X0)))) \quad (22)$$

**Theorem 1**

$$\forall X0.\forall X1.((\neg v2\_struct\_0 X1)\wedge((v1\_group\_1 X1)\wedge(l3\_algstr\_0 X1))\Rightarrow(\forall X2.((\neg v2\_struct\_0 X2)\wedge(m2\_monoid\_0 X2 X1))\Rightarrow((k4\_binop\_1 (u1\_struct\_0 X1) (u2\_algstr\_0 X1) \in u1\_struct\_0 X2)\Rightarrow(m3\_monoid\_0 (k9\_monoid\_1 X2 X0) (k11\_monoid\_1 X1 X0))))))$$