

## t32\_closure2 (TMcBnwendNDYRo- zoAKDzaL7jJe9AQHTuiZd)

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Let  $v1\_relat\_1 : \iota \Rightarrow o$  be given. Let  $v4\_relat\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v1\_funct\_1 : \iota \Rightarrow o$  be given. Let  $v1\_partfun1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $m1\_closure2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k2\_closure2 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v1\_funct\_2 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k6\_closure2 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k1\_zfmisc\_1 : \iota \Rightarrow \iota$  be given. Let  $k2\_zfmisc\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $r6\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k3\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v8\_closure2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $r2\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k7\_closure2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v1\_xboole\_0 : \iota \Rightarrow o$  be given. Let  $k1\_closure2 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v4\_funct\_1 : \iota \Rightarrow o$  be given. Let  $v2\_card\_3 : \iota \Rightarrow o$  be given. Let  $m3\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v1\_closure2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v2\_closure2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v3\_closure2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v4\_closure2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v5\_closure2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v6\_closure2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Assume the following.

$$\begin{aligned}
& \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\
& (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1\_relat\_1 \\
& X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\
& (\forall X3. ((v1\_relat\_1 X3) \wedge ((v4\_relat\_1 X3 X0) \wedge ((v1\_funct\_1 \\
& X3) \wedge (v1\_partfun1 X3 X0)))) \Rightarrow (((r2\_pboole X0 X1 X2) \wedge (r2\_pboole \\
& X0 X1 X3)) \Rightarrow (r2\_pboole X0 X1 (k3\_pboole X0 X2 X3))))))
\end{aligned} \tag{1}$$

Assume the following.

$$\begin{aligned}
& \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\
& (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1\_relat\_1 \\
& X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\
& (r2\_pboole X0 (k3\_pboole X0 X1 X2) X1))
\end{aligned} \tag{2}$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(((v1\_relat\_1 X1)\wedge((v4\_relat\_1 \\ & X1 X0)\wedge((v1\_funct\_1 X1)\wedge(v1\_partfun1 X1 X0))))\wedge((v1\_relat\_1 \\ & X2)\wedge((v4\_relat\_1 X2 X0)\wedge((v1\_funct\_1 X2)\wedge(v1\_partfun1 X2 X0))))\Rightarrow \\ & ((r6\_pboole X0 X1 X2)\Leftrightarrow(X1 = X2)) \end{aligned} \quad (3)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(((v1\_relat\_1 X1)\wedge((v4\_relat\_1 \\ & X1 X0)\wedge((v1\_funct\_1 X1)\wedge(v1\_partfun1 X1 X0))))\wedge((\neg v1\_xboole\_0 \\ & X2)\wedge(m1\_subset\_1 X2 (k1\_zfmisc\_1 (k1\_closure2 X0 X1))))\Rightarrow(\forall X3. \\ & (m1\_closure2 X3 X0 X1 X2)\Leftrightarrow(m1\_subset\_1 X3 X2)) \end{aligned} \quad (4)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.((v1\_relat\_1 X1)\wedge((v4\_relat\_1 X1 X0)\wedge( \\ & (v1\_funct\_1 X1)\wedge(v1\_partfun1 X1 X0))))\Rightarrow(k6\_closure2 X0 X1 = k1\_closure2 \\ & X0 X1) \end{aligned} \quad (5)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.((v1\_relat\_1 X1)\wedge((v4\_relat\_1 X1 X0)\wedge( \\ & (v1\_funct\_1 X1)\wedge(v1\_partfun1 X1 X0))))\Rightarrow(k2\_closure2 X0 X1 = k1\_closure2 \\ & X0 X1) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.((v1\_relat\_1 X1)\wedge((v4\_relat\_1 X1 X0)\wedge( \\ & (v1\_funct\_1 X1)\wedge(v1\_partfun1 X1 X0))))\Rightarrow((\neg v1\_xboole\_0 (k1\_closure2 \\ & X0 X1))\wedge((v4\_funct\_1 (k1\_closure2 X0 X1))\wedge(v2\_card\_3 (k1\_closure2 \\ & X0 X1)))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.((v1\_relat\_1 X1)\wedge((v4\_relat\_1 X1 X0)\wedge( \\ & (v1\_funct\_1 X1)\wedge(v1\_partfun1 X1 X0))))\Rightarrow(\forall X2.(m3\_pboole \\ & X2 X0 X1)\Rightarrow((v1\_relat\_1 X2)\wedge((v4\_relat\_1 X2 X0)\wedge((v1\_funct\_1 X2)\wedge \\ & (v1\_partfun1 X2 X0)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(((v1\_relat\_1 X1)\wedge((v4\_relat\_1 \\ & X1 X0)\wedge((v1\_funct\_1 X1)\wedge(v1\_partfun1 X1 X0))))\wedge((\neg v1\_xboole\_0 \\ & X2)\wedge(m1\_subset\_1 X2 (k1\_zfmisc\_1 (k1\_closure2 X0 X1))))\Rightarrow(\forall X3. \\ & (m1\_closure2 X3 X0 X1 X2)\Rightarrow(m3\_pboole X3 X0 X1)) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.\forall X3.(((v1\_relat\_1 X1)\wedge \\ & ((v4\_relat\_1 X1 X0)\wedge((v1\_funct\_1 X1)\wedge(v1\_partfun1 X1 X0))))\wedge \\ & (((v1\_funct\_1 X2)\wedge((v1\_funct\_2 X2 (k6\_closure2 X0 X1) (k6\_closure2 \\ & X0 X1))\wedge(m1\_subset\_1 X2 (k1\_zfmisc\_1 (k2\_zfmisc\_1 (k6\_closure2 \\ & X0 X1) (k6\_closure2 X0 X1))))))\wedge(m1\_subset\_1 X3 (k6\_closure2 X0 \\ & X1)))\Rightarrow(m1\_closure2 (k7\_closure2 X0 X1 X2 X3) X0 X1 (k6\_closure2 \\ & X0 X1)) \end{aligned} \tag{10}$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.(((v1\_relat\_1 X1)\wedge((v4\_relat\_1 X1 X0)\wedge( \\ & (v1\_funct\_1 X1)\wedge(v1\_partfun1 X1 X0))))\Rightarrow((v1\_closure2 (k6\_closure2 \\ & X0 X1) X0 X1)\wedge((v2\_closure2 (k6\_closure2 X0 X1) X0 X1)\wedge((v3\_closure2 \\ & (k6\_closure2 X0 X1) X0 X1)\wedge((v4\_closure2 (k6\_closure2 X0 X1) X0 \\ & X1)\wedge((v5\_closure2 (k6\_closure2 X0 X1) X0 X1)\wedge((v6\_closure2 (k6\_closure2 \\ & X0 X1) X0 X1)\wedge(m1\_subset\_1 (k6\_closure2 X0 X1) (k1\_zfmisc\_1 (k1\_closure2 \\ & X0 X1)))))))))) \end{aligned} \tag{11}$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(((v1\_relat\_1 X1)\wedge((v4\_relat\_1 \\ & X1 X0)\wedge((v1\_funct\_1 X1)\wedge(v1\_partfun1 X1 X0))))\wedge((v1\_relat\_1 \\ & X2)\wedge((v4\_relat\_1 X2 X0)\wedge((v1\_funct\_1 X2)\wedge(v1\_partfun1 X2 X0))))\Rightarrow \\ & ((v1\_relat\_1 (k3\_pboole X0 X1 X2)\wedge((v4\_relat\_1 (k3\_pboole X0 \\ & X1 X2) X0)\wedge((v1\_funct\_1 (k3\_pboole X0 X1 X2)\wedge(v1\_partfun1 (k3\_pboole \\ & X0 X1 X2) X0)))))) \end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.(((v1\_relat\_1 X1)\wedge((v4\_relat\_1 X1 X0)\wedge( \\ & (v1\_funct\_1 X1)\wedge(v1\_partfun1 X1 X0))))\Rightarrow(\forall X2.((v1\_funct\_1 \\ & X2)\wedge((v1\_funct\_2 X2 (k6\_closure2 X0 X1) (k6\_closure2 X0 X1))\wedge( \\ & m1\_subset\_1 X2 (k1\_zfmisc\_1 (k2\_zfmisc\_1 (k6\_closure2 X0 X1) ( \\ & k6\_closure2 X0 X1))))))\Rightarrow((v8\_closure2 X2 X0 X1)\Leftrightarrow(\forall X3.( \\ & m1\_closure2 X3 X0 X1 (k6\_closure2 X0 X1))\Rightarrow(\forall X4.(m1\_closure2 \\ & X4 X0 X1 (k6\_closure2 X0 X1))\Rightarrow((r2\_pboole X0 X3 X4)\Rightarrow(r2\_pboole X0 \\ & (k7\_closure2 X0 X1 X2 X3) (k7\_closure2 X0 X1 X2 X4)))))) \end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(((v1\_relat\_1 X1)\wedge((v4\_relat\_1 \\ & X1 X0)\wedge((v1\_funct\_1 X1)\wedge(v1\_partfun1 X1 X0))))\wedge((v1\_relat\_1 \\ & X2)\wedge((v4\_relat\_1 X2 X0)\wedge((v1\_funct\_1 X2)\wedge(v1\_partfun1 X2 X0))))\Rightarrow \\ & (k3\_pboole X0 X1 X2 = k3\_pboole X0 X2 X1) \end{aligned} \tag{14}$$

**Theorem 1**

$$\begin{aligned} & \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\ & (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (\forall X2. (m1\_closure2 \\ & X2 X0 X1 (k2\_closure2 X0 X1)) \Rightarrow (\forall X3. (m1\_closure2 X3 X0 X1 ( \\ & k2\_closure2 X0 X1)) \Rightarrow (\forall X4. ((v1\_funct\_1 X4) \wedge ((v1\_funct\_2 \\ & X4 (k6\_closure2 X0 X1) (k6\_closure2 X0 X1)) \wedge (m1\_subset\_1 X4 (k1\_zfmisc\_1 \\ & (k2\_zfmisc\_1 (k6\_closure2 X0 X1) (k6\_closure2 X0 X1)))))) \Rightarrow (\forall X5. \\ & (m1\_closure2 X5 X0 X1 (k6\_closure2 X0 X1)) \Rightarrow (((r6\_pboole X0 X5 (k3\_pboole \\ & X0 X2 X3)) \wedge (v8\_closure2 X4 X0 X1)) \Rightarrow (r2\_pboole X0 (k7\_closure2 X0 \\ & X1 X4 X5) (k3\_pboole X0 (k7\_closure2 X0 X1 X4 X2) (k7\_closure2 X0 X1 \\ & X4 X3))))))))) \end{aligned}$$