

t32_pboole (TMZad- HDBv5gm5aE8HSERqEtnwS5US282BUh)

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Let $v1_relat_1 : \iota \Rightarrow o$ be given. Let $v4_relat_1 : \iota \Rightarrow \iota \Rightarrow o$ be given. Let $v1_funct_1 : \iota \Rightarrow o$ be given. Let $v1_partfun1 : \iota \Rightarrow \iota \Rightarrow o$ be given. Let $r6_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$ be given. Let $k3_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Let $k2_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Let $k3_xboole_0 : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Let $k2_xboole_0 : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Let $k1_funct_1 : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall X0. \forall X1. \forall X2. k3_xboole_0 X0 (k2_xboole_0 X1 X2) = k2_xboole_0 (k3_xboole_0 X0 X1) (k3_xboole_0 X0 X2) \quad (1)$$

Assume the following.

$$\forall X0. \forall X1. \forall X2. (((v1_relat_1 X1) \wedge ((v4_relat_1 X1 X0) \wedge ((v1_funct_1 X1) \wedge (v1_partfun1 X1 X0)))) \wedge ((v1_relat_1 X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))))) \Rightarrow (k2_pboole X0 X1 X1 = X1) \quad (2)$$

Assume the following.

$$\forall X0. \forall X1. \forall X2. (((v1_relat_1 X1) \wedge ((v4_relat_1 X1 X0) \wedge ((v1_funct_1 X1) \wedge (v1_partfun1 X1 X0)))) \wedge ((v1_relat_1 X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))))) \Rightarrow ((v1_relat_1 (k3_pboole X0 X1 X2)) \wedge ((v4_relat_1 (k3_pboole X0 X1 X2) X0) \wedge ((v1_funct_1 (k3_pboole X0 X1 X2)) \wedge (v1_partfun1 (k3_pboole X0 X1 X2) X0)))) \quad (3)$$

Assume the following.

$$\forall X0. \forall X1. \forall X2. (((v1_relat_1 X1) \wedge ((v4_relat_1 X1 X0) \wedge ((v1_funct_1 X1) \wedge (v1_partfun1 X1 X0)))) \wedge ((v1_relat_1 X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))))) \Rightarrow ((v1_relat_1 (k2_pboole X0 X1 X2)) \wedge ((v4_relat_1 (k2_pboole X0 X1 X2) X0) \wedge ((v1_funct_1 (k2_pboole X0 X1 X2)) \wedge (v1_partfun1 (k2_pboole X0 X1 X2) X0)))) \quad (4)$$

Assume the following.

$$\begin{aligned}
& \forall X0.\forall X1.((v1_relat_1 X1)\wedge((v4_relat_1 X1 X0)\wedge \\
& (v1_funct_1 X1)\wedge(v1_partfun1 X1 X0)))\Rightarrow(\forall X2.((v1_relat_1 \\
& X2)\wedge((v4_relat_1 X2 X0)\wedge((v1_funct_1 X2)\wedge(v1_partfun1 X2 X0))))\Rightarrow \\
& (\forall X3.((v1_relat_1 X3)\wedge((v4_relat_1 X3 X0)\wedge((v1_funct_1 \\
& X3)\wedge(v1_partfun1 X3 X0))))\Rightarrow((X3 = k3_pboole X0 X1 X2)\Leftrightarrow(\forall X4. \\
& (X4 \in X0)\Rightarrow(k1_funct_1 X3 X4 = k3_xboole.0 (k1_funct_1 X1 X4) (k1_funct_1 \\
& X2 X4))))))
\end{aligned} \tag{5}$$

Assume the following.

$$\begin{aligned}
& \forall X0.\forall X1.((v1_relat_1 X1)\wedge((v4_relat_1 X1 X0)\wedge(\\
& (v1_funct_1 X1)\wedge(v1_partfun1 X1 X0))))\Rightarrow(\forall X2.((v1_relat_1 \\
& X2)\wedge((v4_relat_1 X2 X0)\wedge((v1_funct_1 X2)\wedge(v1_partfun1 X2 X0))))\Rightarrow \\
& (\forall X3.((v1_relat_1 X3)\wedge((v4_relat_1 X3 X0)\wedge((v1_funct_1 \\
& X3)\wedge(v1_partfun1 X3 X0))))\Rightarrow((X3 = k2_pboole X0 X1 X2)\Leftrightarrow(\forall X4. \\
& (X4 \in X0)\Rightarrow(k1_funct_1 X3 X4 = k2_xboole.0 (k1_funct_1 X1 X4) (k1_funct_1 \\
& X2 X4))))))
\end{aligned} \tag{6}$$

Assume the following.

$$\begin{aligned}
& \forall X0.\forall X1.((v1_relat_1 X1)\wedge((v4_relat_1 X1 X0)\wedge(\\
& (v1_funct_1 X1)\wedge(v1_partfun1 X1 X0))))\Rightarrow(\forall X2.((v1_relat_1 \\
& X2)\wedge((v4_relat_1 X2 X0)\wedge((v1_funct_1 X2)\wedge(v1_partfun1 X2 X0))))\Rightarrow \\
& ((r6_pboole X0 X1 X2)\Leftrightarrow(\forall X3.(X3 \in X0)\Rightarrow(k1_funct_1 X1 X3 = k1_funct_1 \\
& X2 X3))))
\end{aligned} \tag{7}$$

Assume the following.

$$\begin{aligned}
& \forall X0.\forall X1.\forall X2.(((v1_relat_1 X1)\wedge((v4_relat_1 \\
& X1 X0)\wedge((v1_funct_1 X1)\wedge(v1_partfun1 X1 X0))))\wedge((v1_relat_1 \\
& X2)\wedge((v4_relat_1 X2 X0)\wedge((v1_funct_1 X2)\wedge(v1_partfun1 X2 X0))))\Rightarrow \\
& (k3_pboole X0 X1 X2 = k3_pboole X0 X2 X1)
\end{aligned} \tag{8}$$

Assume the following.

$$\begin{aligned}
& \forall X0.\forall X1.\forall X2.(((v1_relat_1 X1)\wedge((v4_relat_1 \\
& X1 X0)\wedge((v1_funct_1 X1)\wedge(v1_partfun1 X1 X0))))\wedge((v1_relat_1 \\
& X2)\wedge((v4_relat_1 X2 X0)\wedge((v1_funct_1 X2)\wedge(v1_partfun1 X2 X0))))\Rightarrow \\
& (k2_pboole X0 X1 X2 = k2_pboole X0 X2 X1)
\end{aligned} \tag{9}$$

Theorem 1

$$\begin{aligned} & \forall X0. \forall X1. ((v1_relat_1 X1) \wedge ((v4_relat_1 X1 X0) \wedge \\ & (v1_funct_1 X1) \wedge (v1_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1_relat_1 \\ & X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))) \Rightarrow \\ & (\forall X3. ((v1_relat_1 X3) \wedge ((v4_relat_1 X3 X0) \wedge ((v1_funct_1 \\ & X3) \wedge (v1_partfun1 X3 X0)))) \Rightarrow (r6_pboole X0 (k3_pboole X0 X1 (k2_pboole \\ & X0 X2 X3)) (k2_pboole X0 (k3_pboole X0 X1 X2) (k3_pboole X0 X1 X3)))))) \end{aligned}$$