

# t33\_boolealg (TMFTpFgxSAyy- hHZ5Rp9shB34TgndTDFsgyR)

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Let  $v2\_struct\_0 : \iota \Rightarrow o$  be given. Let  $v10\_lattices : \iota \Rightarrow o$  be given. Let  $v17\_lattices : \iota \Rightarrow o$  be given. Let  $l3\_lattices : \iota \Rightarrow o$  be given. Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $u1\_struct\_0 : \iota \Rightarrow \iota$  be given. Let  $r1\_boolealg : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k1\_boolealg : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k4\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v6\_lattices : \iota \Rightarrow o$  be given. Let  $v7\_lattices : \iota \Rightarrow o$  be given. Let  $v8\_lattices : \iota \Rightarrow o$  be given. Let  $v9\_lattices : \iota \Rightarrow o$  be given. Let  $r3\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $r1\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k7\_lattices : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k5\_lattices : \iota \Rightarrow \iota$  be given. Let  $k3\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $l1\_lattices : \iota \Rightarrow o$  be given. Let  $l2\_lattices : \iota \Rightarrow o$  be given. Let  $v4\_lattices : \iota \Rightarrow o$  be given. Let  $v5\_lattices : \iota \Rightarrow o$  be given. Assume the following.

$$\begin{aligned}
& \forall X0. ((\neg v2\_struct\_0 X0) \wedge ((v6\_lattices X0) \wedge ((v7\_lattices \\
& X0) \wedge ((v8\_lattices X0) \wedge ((v9\_lattices X0) \wedge (l3\_lattices X0)))))) \Rightarrow \\
& (\forall X1. (m1\_subset\_1 X1 (u1\_struct\_0 X0)) \Rightarrow (\forall X2. (m1\_subset\_1 \\
& X2 (u1\_struct\_0 X0)) \Rightarrow (\forall X3. (m1\_subset\_1 X3 (u1\_struct\_0 \\
& X0)) \Rightarrow (((r3\_lattices X0 X1 X2) \wedge (r3\_lattices X0 X1 X3)) \Rightarrow (r3\_lattices \\
& X0 X1 (k4\_lattices X0 X2 X3))))))
\end{aligned} \tag{1}$$

Assume the following.

$$\begin{aligned}
& \forall X0. ((\neg v2\_struct\_0 X0) \wedge ((v6\_lattices X0) \wedge ((v8\_lattices \\
& X0) \wedge (l3\_lattices X0)))) \Rightarrow (\forall X1. (m1\_subset\_1 X1 (u1\_struct\_0 \\
& X0)) \Rightarrow (\forall X2. (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (r1\_lattices \\
& X0 (k4\_lattices X0 X1 X2) X1)))
\end{aligned} \tag{2}$$

Assume the following.

$$\begin{aligned}
& \forall X0. ((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge ((v17\_lattices \\
& X0) \wedge (l3\_lattices X0)))) \Rightarrow (\forall X1. (m1\_subset\_1 X1 (u1\_struct\_0 \\
& X0)) \Rightarrow (\forall X2. (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow ((r3\_lattices \\
& X0 X1 X2) \Leftrightarrow (k4\_lattices X0 X1 (k7\_lattices X0 X2) = k5\_lattices X0))))
\end{aligned} \tag{3}$$

Assume the following.

$$\begin{aligned} \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices \\ X0))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 X0)) \Rightarrow (\forall X2. \\ (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (\forall X3.(m1\_subset\_1 X3 \\ (u1\_struct\_0 X0)) \Rightarrow ((r3\_lattices X0 X1 X2) \Rightarrow (r3\_lattices X0 (k1\_boolealg \\ X0 X1 X3) X2)))))) \end{aligned} \quad (4)$$

Assume the following.

$$\begin{aligned} \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices \\ X0))) \Rightarrow (((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge ((v17\_lattices \\ X0) \wedge (l3\_lattices X0)))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 \\ X0)) \Rightarrow (\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow ((r3\_lattices \\ X0 (k4\_lattices X0 X1 (k3\_lattices X0 (k7\_lattices X0 X1) X2)) X2) \wedge \\ (\forall X3.(m1\_subset\_1 X3 (u1\_struct\_0 X0)) \Rightarrow ((r3\_lattices \\ X0 (k4\_lattices X0 X1 X3) X2) \Rightarrow (r3\_lattices X0 X3 (k3\_lattices X0 \\ (k7\_lattices X0 X1) X2)))))))))) \end{aligned} \quad (5)$$

Assume the following.

$$\begin{aligned} \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge ((v17\_lattices \\ X0) \wedge (l3\_lattices X0)))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 \\ X0)) \Rightarrow (\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow ((k4\_lattices \\ X0 X1 X2 = k5\_lattices X0) \Leftrightarrow (r3\_lattices X0 X1 (k7\_lattices X0 X2)))))) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge ((v17\_lattices \\ X0) \wedge (l3\_lattices X0)))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 \\ X0)) \Rightarrow (\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (k7\_lattices \\ X0 (k4\_lattices X0 X1 X2) = k3\_lattices X0 (k7\_lattices X0 X1) (k7\_lattices \\ X0 X2)))))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge ((v17\_lattices \\ X0) \wedge (l3\_lattices X0)))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 \\ X0)) \Rightarrow (\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (\forall X3. \\ (m1\_subset\_1 X3 (u1\_struct\_0 X0)) \Rightarrow ((r3\_lattices X0 X1 X2) \Rightarrow (r3\_lattices \\ X0 (k1\_boolealg X0 X3 X2) (k1\_boolealg X0 X3 X1)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} \forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0) \wedge ((v6\_lattices \\ X0) \wedge ((v8\_lattices X0) \wedge ((v9\_lattices X0) \wedge (l3\_lattices X0)))) \wedge \\ ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge (m1\_subset\_1 X2 (u1\_struct\_0 \\ X0)))) \Rightarrow (r3\_lattices X0 X1 X1) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((\neg v2\_struct\_0 X0) \wedge (v6\_lattices \\ & X0) \wedge (v8\_lattices X0) \wedge (v9\_lattices X0) \wedge (l3\_lattices X0)))) \wedge \\ & ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge (m1\_subset\_1 X2 (u1\_struct\_0 \\ & X0))) \Rightarrow ((r3\_lattices X0 X1 X2) \Leftrightarrow (r1\_lattices X0 X1 X2)) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. (((\neg v2\_struct\_0 X0) \wedge (v10\_lattices X0) \wedge \\ & (v17\_lattices X0) \wedge (l3\_lattices X0)))) \wedge (m1\_subset\_1 X1 (u1\_struct\_0 \\ & X0)) \Rightarrow (k7\_lattices X0 (k7\_lattices X0 X1) = X1) \end{aligned} \quad (11)$$

Assume the following.

$$\forall X0. (l3\_lattices X0) \Rightarrow ((l1\_lattices X0) \wedge (l2\_lattices X0)) \quad (12)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. (((\neg v2\_struct\_0 X0) \wedge (l3\_lattices X0)) \wedge \\ & (m1\_subset\_1 X1 (u1\_struct\_0 X0))) \Rightarrow (m1\_subset\_1 (k7\_lattices \\ & X0 X1) (u1\_struct\_0 X0)) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((\neg v2\_struct\_0 X0) \wedge (v6\_lattices \\ & X0) \wedge (l1\_lattices X0))) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge \\ & (m1\_subset\_1 X2 (u1\_struct\_0 X0))) \Rightarrow (m1\_subset\_1 (k4\_lattices \\ & X0 X1 X2) (u1\_struct\_0 X0)) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((\neg v2\_struct\_0 X0) \wedge (v10\_lattices \\ & X0) \wedge (l3\_lattices X0))) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge \\ & (m1\_subset\_1 X2 (u1\_struct\_0 X0))) \Rightarrow (m1\_subset\_1 (k1\_boolealg \\ & X0 X1 X2) (u1\_struct\_0 X0)) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} & \forall X0. ((\neg v2\_struct\_0 X0) \wedge (v10\_lattices X0) \wedge (l3\_lattices \\ & X0)) \Rightarrow (\forall X1. (m1\_subset\_1 X1 (u1\_struct\_0 X0)) \Rightarrow (\forall X2. \\ & (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow ((r1\_boolealg X0 X1 X2) \Leftrightarrow ((r3\_lattices \\ & X0 X1 X2) \wedge (r3\_lattices X0 X2 X1)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} & \forall X0. ((\neg v2\_struct\_0 X0) \wedge (v10\_lattices X0) \wedge (l3\_lattices \\ & X0)) \Rightarrow (\forall X1. (m1\_subset\_1 X1 (u1\_struct\_0 X0)) \Rightarrow (\forall X2. \\ & (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (k1\_boolealg X0 X1 X2 = k4\_lattices \\ & X0 X1 (k7\_lattices X0 X2)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} \forall X0. \forall X1. \forall X2. (&((\neg v2\_struct\_0 X0) \wedge (v6\_lattices \\ &X0) \wedge (l1\_lattices X0))) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge ( \\ &m1\_subset\_1 X2 (u1\_struct\_0 X0))) \Rightarrow (k4\_lattices X0 X1 X2 = k4\_lattices \\ &X0 X2 X1) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} \forall X0. (l3\_lattices X0) \Rightarrow (&((\neg v2\_struct\_0 X0) \wedge (v10\_lattices \\ &X0)) \Rightarrow ((\neg v2\_struct\_0 X0) \wedge (v4\_lattices X0) \wedge (v5\_lattices X0) \wedge \\ &((v6\_lattices X0) \wedge (v7\_lattices X0) \wedge (v8\_lattices X0) \wedge (v9\_lattices \\ &X0)))))) \end{aligned} \quad (19)$$

**Theorem 1**

$$\begin{aligned} \forall X0. (&((\neg v2\_struct\_0 X0) \wedge (v10\_lattices X0) \wedge (v17\_lattices \\ &X0) \wedge (l3\_lattices X0))) \Rightarrow (\forall X1. (m1\_subset\_1 X1 (u1\_struct\_0 \\ &X0)) \Rightarrow (\forall X2. (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (r1\_boolealg \\ &X0 (k1\_boolealg X0 X1 (k4\_lattices X0 X1 X2)) (k1\_boolealg X0 X1 X2)))) \end{aligned}$$