

t34\_mssubfam  
(TMNHqT2RnbDyudSeMerkXspgD3ivK7xTrY8)

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Let  $v1\_relat\_1 : \iota \Rightarrow o$  be given. Let  $v4\_relat\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v1\_funct\_1 : \iota \Rightarrow o$  be given. Let  $v1\_partfun1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $m3\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k1\_mboolean : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k2\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $r2\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\ & (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1\_relat\_1 \\ & X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\ & (\forall X3. ((v1\_relat\_1 X3) \wedge ((v4\_relat\_1 X3 X0) \wedge ((v1\_funct\_1 \\ & X3) \wedge (v1\_partfun1 X3 X0)))) \Rightarrow (((r2\_pboole X0 X1 X2) \wedge (r2\_pboole \\ & X0 X3 X2)) \Rightarrow (r2\_pboole X0 (k2\_pboole X0 X1 X3) X2))) \end{aligned} \quad (1)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\ & (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (\forall X2. (m3\_pboole \\ & X2 X0 X1) \Rightarrow ((v1\_relat\_1 X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge \\ & (v1\_partfun1 X2 X0)))) \end{aligned} \quad (2)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 \\ & X1 X0) \wedge ((v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0)))) \wedge ((v1\_relat\_1 \\ & X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\ & ((v1\_relat\_1 (k2\_pboole X0 X1 X2)) \wedge ((v4\_relat\_1 (k2\_pboole X0 \\ & X1 X2) X0) \wedge ((v1\_funct\_1 (k2\_pboole X0 X1 X2)) \wedge (v1\_partfun1 (k2\_pboole \\ & X0 X1 X2) X0)))) \end{aligned} \quad (3)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\ & (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow ((v1\_relat\_1 (k1\_mboolean \\ & X0 X1)) \wedge ((v4\_relat\_1 (k1\_mboolean X0 X1) X0) \wedge ((v1\_funct\_1 (k1\_mboolean \\ & X0 X1)) \wedge (v1\_partfun1 (k1\_mboolean X0 X1) X0)))) \end{aligned} \quad (4)$$

Assume the following.

$$\begin{aligned}
& \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\
& (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1\_relat\_1 \\
& X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\
& ((m3\_pboole X2 X0 X1) \Leftrightarrow (r2\_pboole X0 X2 X1)))
\end{aligned} \tag{5}$$

**Theorem 1**

$$\begin{aligned}
& \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\
& (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (\forall X2. (m3\_pboole \\
& X2 X0 (k1\_mboolean X0 X1)) \Rightarrow (\forall X3. (m3\_pboole X3 X0 (k1\_mboolean \\
& X0 X1)) \Rightarrow (m3\_pboole (k2\_pboole X0 X2 X3) X0 (k1\_mboolean X0 X1))))
\end{aligned}$$