

t36\_boolealg  
(TMY4LXCkWuADqwxXaufESHALaDnxFeezhdY)

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Let  $v2\_struct\_0 : \iota \Rightarrow o$  be given. Let  $v10\_lattices : \iota \Rightarrow o$  be given. Let  $v17\_lattices : \iota \Rightarrow o$  be given. Let  $l3\_lattices : \iota \Rightarrow o$  be given. Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $u1\_struct\_0 : \iota \Rightarrow \iota$  be given. Let  $r1\_boolealg : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k3\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k4\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_boolealg : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k7\_lattices : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k6\_lattices : \iota \Rightarrow \iota$  be given. Let  $v6\_lattices : \iota \Rightarrow o$  be given. Let  $l1\_lattices : \iota \Rightarrow o$  be given. Let  $k2\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v4\_lattices : \iota \Rightarrow o$  be given. Let  $l2\_lattices : \iota \Rightarrow o$  be given. Let  $k1\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v14\_lattices : \iota \Rightarrow o$  be given. Let  $v11\_lattices : \iota \Rightarrow o$  be given. Let  $v15\_lattices : \iota \Rightarrow o$  be given. Let  $v16\_lattices : \iota \Rightarrow o$  be given. Let  $v13\_lattices : \iota \Rightarrow o$  be given. Let  $v5\_lattices : \iota \Rightarrow o$  be given. Let  $v7\_lattices : \iota \Rightarrow o$  be given. Let  $v8\_lattices : \iota \Rightarrow o$  be given. Let  $v9\_lattices : \iota \Rightarrow o$  be given. Assume the following.

$$\begin{aligned} \forall X0. ((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge ((v17\_lattices \\ X0) \wedge (l3\_lattices X0)))) \Rightarrow (\forall X1. (m1\_subset\_1 X1 (u1\_struct\_0 \\ X0)) \Rightarrow (k3\_lattices X0 (k7\_lattices X0 X1) X1 = k6\_lattices X0)) \end{aligned} \quad (1)$$

Assume the following.

$$\begin{aligned} \forall X0. \forall X1. \forall X2. (((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices \\ X0) \wedge (l3\_lattices X0))) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge ( \\ m1\_subset\_1 X2 (u1\_struct\_0 X0)))) \Rightarrow ((r1\_boolealg X0 X1 X2) \Leftrightarrow (X1 = \\ X2)) \end{aligned} \quad (2)$$

Assume the following.

$$\begin{aligned} \forall X0. \forall X1. \forall X2. (((\neg v2\_struct\_0 X0) \wedge ((v6\_lattices \\ X0) \wedge (l1\_lattices X0))) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge ( \\ m1\_subset\_1 X2 (u1\_struct\_0 X0)))) \Rightarrow (k4\_lattices X0 X1 X2 = k2\_lattices \\ X0 X1 X2) \end{aligned} \quad (3)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge(v4\_lattices X0)\wedge(l2\_lattices X0)))\wedge((m1\_subset\_1 X1 (u1\_struct\_0 X0))\wedge(m1\_subset\_1 X2 (u1\_struct\_0 X0)))\Rightarrow(k3\_lattices X0 X1 X2 = k1\_lattices X0 X1 X2) \quad (4)$$

Assume the following.

$$\forall X0.\forall X1.(((\neg v2\_struct\_0 X0)\wedge(v10\_lattices X0)\wedge(v14\_lattices X0)\wedge(l3\_lattices X0)))\wedge(m1\_subset\_1 X1 (u1\_struct\_0 X0))\Rightarrow(k4\_lattices X0 (k6\_lattices X0) X1 = X1) \quad (5)$$

Assume the following.

$$\forall X0.(l3\_lattices X0)\Rightarrow((l1\_lattices X0)\wedge(l2\_lattices X0)) \quad (6)$$

Assume the following.

$$\forall X0.\forall X1.(((\neg v2\_struct\_0 X0)\wedge(l3\_lattices X0))\wedge(m1\_subset\_1 X1 (u1\_struct\_0 X0)))\Rightarrow(m1\_subset\_1 (k7\_lattices X0 X1) (u1\_struct\_0 X0)) \quad (7)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0)\wedge(l2\_lattices X0))\Rightarrow(m1\_subset\_1 (k6\_lattices X0) (u1\_struct\_0 X0)) \quad (8)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge(v6\_lattices X0)\wedge(l1\_lattices X0)))\wedge((m1\_subset\_1 X1 (u1\_struct\_0 X0))\wedge(m1\_subset\_1 X2 (u1\_struct\_0 X0)))\Rightarrow(m1\_subset\_1 (k4\_lattices X0 X1 X2) (u1\_struct\_0 X0)) \quad (9)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge(v4\_lattices X0)\wedge(l2\_lattices X0)))\wedge((m1\_subset\_1 X1 (u1\_struct\_0 X0))\wedge(m1\_subset\_1 X2 (u1\_struct\_0 X0)))\Rightarrow(m1\_subset\_1 (k3\_lattices X0 X1 X2) (u1\_struct\_0 X0)) \quad (10)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge(v10\_lattices X0)\wedge(l3\_lattices X0)))\wedge((m1\_subset\_1 X1 (u1\_struct\_0 X0))\wedge(m1\_subset\_1 X2 (u1\_struct\_0 X0)))\Rightarrow(m1\_subset\_1 (k1\_boolealg X0 X1 X2) (u1\_struct\_0 X0)) \quad (11)$$

Assume the following.

$$\begin{aligned} & \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices \\ & X0))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 X0)) \Rightarrow (\forall X2. \\ & (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (k1\_boolealg X0 X1 X2 = k4\_lattices \\ & X0 X1 (k7\_lattices X0 X2)))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} & \forall X0.((\neg v2\_struct\_0 X0) \wedge (l3\_lattices X0)) \Rightarrow ((v11\_lattices \\ & X0) \Leftrightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 X0)) \Rightarrow (\forall X2. \\ & (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (\forall X3.(m1\_subset\_1 X3 \\ & (u1\_struct\_0 X0)) \Rightarrow (k2\_lattices X0 X1 (k1\_lattices X0 X2 X3) = k1\_lattices \\ & X0 (k2\_lattices X0 X1 X2) (k2\_lattices X0 X1 X3)))))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0) \wedge ((v6\_lattices \\ & X0) \wedge (l1\_lattices X0))) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge ( \\ & m1\_subset\_1 X2 (u1\_struct\_0 X0)))) \Rightarrow (k4\_lattices X0 X1 X2 = k4\_lattices \\ & X0 X2 X1) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0) \wedge ((v4\_lattices \\ & X0) \wedge (l2\_lattices X0))) \wedge ((m1\_subset\_1 X1 (u1\_struct\_0 X0)) \wedge ( \\ & m1\_subset\_1 X2 (u1\_struct\_0 X0)))) \Rightarrow (k3\_lattices X0 X1 X2 = k3\_lattices \\ & X0 X2 X1) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} & \forall X0.(l3\_lattices X0) \Rightarrow (((\neg v2\_struct\_0 X0) \wedge (v17\_lattices \\ & X0)) \Rightarrow ((\neg v2\_struct\_0 X0) \wedge ((v11\_lattices X0) \wedge ((v15\_lattices \\ & X0) \wedge (v16\_lattices X0)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} & \forall X0.(l3\_lattices X0) \Rightarrow (((\neg v2\_struct\_0 X0) \wedge (v15\_lattices \\ & X0)) \Rightarrow ((\neg v2\_struct\_0 X0) \wedge ((v13\_lattices X0) \wedge (v14\_lattices X0)))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & \forall X0.(l3\_lattices X0) \Rightarrow (((\neg v2\_struct\_0 X0) \wedge (v10\_lattices \\ & X0)) \Rightarrow ((\neg v2\_struct\_0 X0) \wedge ((v4\_lattices X0) \wedge ((v5\_lattices X0) \wedge \\ & ((v6\_lattices X0) \wedge ((v7\_lattices X0) \wedge ((v8\_lattices X0) \wedge (v9\_lattices \\ & X0)))))))))) \end{aligned} \quad (18)$$

**Theorem 1**

$$\begin{aligned} & \forall X0. ((\neg v2\_struct\_0 X0) \wedge (v10\_lattices X0) \wedge ((v17\_lattices \\ & X0) \wedge (l3\_lattices X0))) \Rightarrow (\forall X1. (m1\_subset\_1 X1 (u1\_struct\_0 \\ & X0)) \Rightarrow (\forall X2. (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (r1\_boolealg \\ & X0 (k3\_lattices X0 (k4\_lattices X0 X1 X2) (k1\_boolealg X0 X1 X2)) \\ & X1))) \end{aligned}$$