

### t3\_closure1

(TMEk5tufPmwkQAm7QUnvmnxpDnW6KEdoLRU)

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Let  $v1\_relat\_1 : \iota \Rightarrow o$  be given. Let  $v4\_relat\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v1\_funct\_1 : \iota \Rightarrow o$  be given. Let  $v1\_partfun1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v2\_relat\_1 : \iota \Rightarrow o$  be given. Let  $m2\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $r1\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k16\_pralg\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $m1\_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Assume the following.

$$\begin{aligned} \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\ (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow ((\exists X2. ((v1\_relat\_1 \\ X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \wedge \\ (r1\_pboole X0 X2 X1)) \Rightarrow (v2\_relat\_1 X1)) \end{aligned} \quad (1)$$

Assume the following.

$$\begin{aligned} \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\ (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1\_relat\_1 \\ X2) \wedge ((v2\_relat\_1 X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge \\ (v1\_partfun1 X2 X0)))))) \Rightarrow ((m1\_pboole X1 X0 X2) \Rightarrow (r1\_pboole X0 X1 X2)) \end{aligned} \quad (2)$$

Assume the following.

$$\begin{aligned} \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\ (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1\_relat\_1 \\ X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\ ((r1\_pboole X0 X1 X2) \Rightarrow (m1\_pboole X1 X0 X2)) \end{aligned} \quad (3)$$

Assume the following.

$$\begin{aligned} \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v2\_relat\_1 X1) \wedge ((v4\_relat\_1 \\ X1 X0) \wedge ((v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0)))))) \Rightarrow (\forall X2. \\ ((v1\_relat\_1 X2) \wedge ((v2\_relat\_1 X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 \\ X2) \wedge (v1\_partfun1 X2 X0)))))) \Rightarrow (\forall X3. (m2\_pboole X3 X0 X1 X2) \Rightarrow \\ (\forall X4. (m1\_pboole X4 X0 X1) \Rightarrow (m1\_pboole (k16\_pralg\_1 X0 X3 \\ X4) X0 X2))) \end{aligned} \quad (4)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\ & (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (\forall X2. (m1\_pboole \\ & X2 X0 X1) \Rightarrow ((v1\_relat\_1 X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge \\ & (v1\_partfun1 X2 X0)))))) \end{aligned} \tag{5}$$

**Theorem 1**

$$\begin{aligned} & \forall X0. \forall X1. ((v1\_relat\_1 X1) \wedge ((v4\_relat\_1 X1 X0) \wedge \\ & (v1\_funct\_1 X1) \wedge (v1\_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1\_relat\_1 \\ & X2) \wedge ((v4\_relat\_1 X2 X0) \wedge ((v1\_funct\_1 X2) \wedge (v1\_partfun1 X2 X0)))) \Rightarrow \\ & (\forall X3. ((v1\_relat\_1 X3) \wedge ((v2\_relat\_1 X3) \wedge ((v4\_relat\_1 \\ & X3 X0) \wedge ((v1\_funct\_1 X3) \wedge (v1\_partfun1 X3 X0)))))) \Rightarrow (\forall X4. \\ & (m2\_pboole X4 X0 X1 X3) \Rightarrow ((r1\_pboole X0 X2 X1) \Rightarrow (r1\_pboole X0 (k16\_pralg\_1 \\ & X0 X4 X2) X3)))) \end{aligned}$$