

t42\_bilinear  
(TMSCDxBmjPs9gL2xdY5qCR5yCEL3CCNjqtq)

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Let  $v2\_struct\_0 : \iota \Rightarrow o$  be given. Let  $v13\_algstr\_0 : \iota \Rightarrow o$  be given. Let  $v3\_rlvect\_1 : \iota \Rightarrow o$  be given. Let  $v4\_rlvect\_1 : \iota \Rightarrow o$  be given. Let  $v1\_vectsp\_1 : \iota \Rightarrow o$  be given. Let  $l6\_algstr\_0 : \iota \Rightarrow o$  be given. Let  $l1\_vectsp\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v1\_funct\_1 : \iota \Rightarrow o$  be given. Let  $v1\_funct\_2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k2\_zfmisc\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $u1\_struct\_0 : \iota \Rightarrow \iota$  be given. Let  $v2\_bilinear : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $v4\_bilinear : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k1\_zfmisc\_1 : \iota \Rightarrow \iota$  be given. Let  $v1\_vectsp\_4 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k10\_bilinear : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k6\_algstr\_0 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k4\_struct\_0 : \iota \Rightarrow \iota$  be given. Let  $l3\_algstr\_0 : \iota \Rightarrow o$  be given. Let  $k2\_binop\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k4\_vectsp\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $l2\_algstr\_0 : \iota \Rightarrow o$  be given. Let  $k1\_algstr\_0 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $l5\_algstr\_0 : \iota \Rightarrow o$  be given. Let  $l4\_algstr\_0 : \iota \Rightarrow o$  be given. Let  $l4\_struct\_0 : \iota \Rightarrow o$  be given. Let  $l3\_struct\_0 : \iota \Rightarrow o$  be given. Let  $l2\_struct\_0 : \iota \Rightarrow o$  be given. Let  $l1\_algstr\_0 : \iota \Rightarrow o$  be given. Let  $l1\_struct\_0 : \iota \Rightarrow o$  be given. Assume the following.

$$\begin{aligned}
& \forall X0. ((\neg v2\_struct\_0 X0) \wedge ((v13\_algstr\_0 X0) \wedge ((v3\_rlvect\_1 \\
& X0) \wedge ((v4\_rlvect\_1 X0) \wedge ((v1\_vectsp\_1 X0) \wedge (l6\_algstr\_0 X0)))))) \Rightarrow \\
& (\forall X1. (m1\_subset\_1 X1 (u1\_struct\_0 X0)) \Rightarrow (k6\_algstr\_0 X0 \\
& X1 (k4\_struct\_0 X0) = k4\_struct\_0 X0)) \tag{1}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall X0.((\neg v2\_struct\_0 X0) \wedge (l3\_algstr\_0 X0)) \Rightarrow (\forall X1. \\
& ((\neg v2\_struct\_0 X1) \wedge (l1\_vectsp\_1 X1 X0)) \Rightarrow (\forall X2.((\neg v2\_struct\_0 \\
& X2) \wedge (l1\_vectsp\_1 X2 X0)) \Rightarrow (\forall X3.(m1\_subset\_1 X3 (u1\_struct\_0 \\
& X1)) \Rightarrow (\forall X4.(m1\_subset\_1 X4 (u1\_struct\_0 X2)) \Rightarrow (\forall X5. \\
& (m1\_subset\_1 X5 (u1\_struct\_0 X0)) \Rightarrow (\forall X6.((v1\_funct\_1 X6) \wedge \\
& ((v1\_funct\_2 X6 (k2\_zfmisc\_1 (u1\_struct\_0 X1) (u1\_struct\_0 X2)) \\
& (u1\_struct\_0 X0)) \wedge (m1\_subset\_1 X6 (k1\_zfmisc\_1 (k2\_zfmisc\_1 \\
& (k2\_zfmisc\_1 (u1\_struct\_0 X1) (u1\_struct\_0 X2)) (u1\_struct\_0 \\
& X0)))))) \Rightarrow ((v4\_bilinear X6 X0 X1 X2) \Rightarrow (k2\_binop\_1 (u1\_struct\_0 \\
& X1) (u1\_struct\_0 X2) (u1\_struct\_0 X0) X6 (k4\_vectsp\_1 X0 X1 X5 X3) \\
& X4 = k6\_algstr\_0 X0 X5 (k2\_binop\_1 (u1\_struct\_0 X1) (u1\_struct\_0 \\
& X2) (u1\_struct\_0 X0) X6 X3 X4)))))))))
\end{aligned} \tag{2}$$

Assume the following.

$$\begin{aligned}
& \forall X0.((\neg v2\_struct\_0 X0) \wedge (l2\_algstr\_0 X0)) \Rightarrow (\forall X1. \\
& ((\neg v2\_struct\_0 X1) \wedge (l1\_vectsp\_1 X1 X0)) \Rightarrow (\forall X2.((\neg v2\_struct\_0 \\
& X2) \wedge (l1\_vectsp\_1 X2 X0)) \Rightarrow (\forall X3.(m1\_subset\_1 X3 (u1\_struct\_0 \\
& X1)) \Rightarrow (\forall X4.(m1\_subset\_1 X4 (u1\_struct\_0 X1)) \Rightarrow (\forall X5. \\
& (m1\_subset\_1 X5 (u1\_struct\_0 X2)) \Rightarrow (\forall X6.((v1\_funct\_1 X6) \wedge \\
& ((v1\_funct\_2 X6 (k2\_zfmisc\_1 (u1\_struct\_0 X1) (u1\_struct\_0 X2)) \\
& (u1\_struct\_0 X0)) \wedge (m1\_subset\_1 X6 (k1\_zfmisc\_1 (k2\_zfmisc\_1 \\
& (k2\_zfmisc\_1 (u1\_struct\_0 X1) (u1\_struct\_0 X2)) (u1\_struct\_0 \\
& X0)))))) \Rightarrow ((v2\_bilinear X6 X0 X1 X2) \Rightarrow (k2\_binop\_1 (u1\_struct\_0 \\
& X1) (u1\_struct\_0 X2) (u1\_struct\_0 X0) X6 (k1\_algstr\_0 X1 X3 X4) X5 = \\
& k1\_algstr\_0 X0 (k2\_binop\_1 (u1\_struct\_0 X1) (u1\_struct\_0 X2) ( \\
& u1\_struct\_0 X0) X6 X3 X5) (k2\_binop\_1 (u1\_struct\_0 X1) (u1\_struct\_0 \\
& X2) (u1\_struct\_0 X0) X6 X4 X5)))))))))
\end{aligned} \tag{3}$$

Assume the following.

$$\forall X0.(l6\_algstr\_0 X0) \Rightarrow ((l2\_algstr\_0 X0) \wedge (l5\_algstr\_0 X0)) \tag{4}$$

Assume the following.

$$\forall X0.(l5\_algstr\_0 X0) \Rightarrow ((l4\_algstr\_0 X0) \wedge (l4\_struct\_0 X0)) \tag{5}$$

Assume the following.

$$\forall X0.(l4\_algstr\_0 X0) \Rightarrow ((l3\_struct\_0 X0) \wedge (l3\_algstr\_0 X0)) \tag{6}$$

Assume the following.

$$\forall X0.(l2\_algstr\_0 X0) \Rightarrow ((l2\_struct\_0 X0) \wedge (l1\_algstr\_0 X0)) \tag{7}$$

Assume the following.

$$\forall X0.(l1\_struct\_0 X0) \Rightarrow (\forall X1.(l1\_vectsp\_1 X1 X0) \Rightarrow (l2\_algstr\_0 X1)) \tag{8}$$

Assume the following.

$$\forall X0.(l1\_algstr\_0 X0) \Rightarrow (l1\_struct\_0 X0) \quad (9)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.\forall X3.(((\neg v2\_struct\_0 \\ & X0) \wedge (l1\_struct\_0 X0)) \wedge (((\neg v2\_struct\_0 X1) \wedge (l1\_vectsp\_1 X1 X0)) \wedge \\ & ((m1\_subset\_1 X2 (u1\_struct\_0 X0)) \wedge (m1\_subset\_1 X3 (u1\_struct\_0 \\ & X1)))))) \Rightarrow (m1\_subset\_1 (k4\_vectsp\_1 X0 X1 X2 X3) (u1\_struct\_0 X1)) \end{aligned} \quad (10)$$

Assume the following.

$$\forall X0.(l2\_struct\_0 X0) \Rightarrow (m1\_subset\_1 (k4\_struct\_0 X0) (u1\_struct\_0 X0)) \quad (11)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.((l1\_algstr\_0 X0) \wedge ((m1\_subset\_1 \\ & X1 (u1\_struct\_0 X0)) \wedge (m1\_subset\_1 X2 (u1\_struct\_0 X0)))) \Rightarrow (m1\_subset\_1 \\ & (k1\_algstr\_0 X0 X1 X2) (u1\_struct\_0 X0)) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.\forall X3.((l2\_struct\_0 X0) \wedge \\ & (((\neg v2\_struct\_0 X1) \wedge (l1\_vectsp\_1 X1 X0)) \wedge (((\neg v2\_struct\_0 X2) \wedge \\ & (l1\_vectsp\_1 X2 X0)) \wedge ((v1\_funct\_1 X3) \wedge ((v1\_funct\_2 X3 (k2\_zfmisc\_1 \\ & (u1\_struct\_0 X1) (u1\_struct\_0 X2)) (u1\_struct\_0 X0)) \wedge (m1\_subset\_1 \\ & X3 (k1\_zfmisc\_1 (k2\_zfmisc\_1 (k2\_zfmisc\_1 (u1\_struct\_0 X1) (u1\_struct\_0 \\ & X2)) (u1\_struct\_0 X0)))))))))) \Rightarrow (m1\_subset\_1 (k10\_bilinear X0 \\ & X1 X2 X3) (k1\_zfmisc\_1 (u1\_struct\_0 X1))) \end{aligned} \quad (13)$$

Assume the following.

$$\forall X0.(l2\_algstr\_0 X0) \Rightarrow ((v4\_rlvect\_1 X0) \Leftrightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 X0) \Rightarrow (k1\_algstr\_0 X0 X1 (k4\_struct\_0 X0) = X1))) \quad (14)$$

Assume the following.

$$\begin{aligned} & \forall X0.((\neg v2\_struct\_0 X0) \wedge (l3\_algstr\_0 X0)) \Rightarrow (\forall X1. \\ & ((\neg v2\_struct\_0 X1) \wedge (l1\_vectsp\_1 X1 X0)) \Rightarrow (\forall X2.(m1\_subset\_1 \\ & X2 (k1\_zfmisc\_1 (u1\_struct\_0 X1))) \Rightarrow ((v1\_vectsp\_4 X2 X0 X1) \Leftrightarrow (( \\ & \forall X3.(m1\_subset\_1 X3 (u1\_struct\_0 X1)) \Rightarrow (\forall X4.(m1\_subset\_1 \\ & X4 (u1\_struct\_0 X1)) \Rightarrow (((X3 \in X2) \wedge (X4 \in X2)) \Rightarrow (k1\_algstr\_0 X1 X3 X4 \in \\ & X2)))))) \wedge (\forall X3.(m1\_subset\_1 X3 (u1\_struct\_0 X0)) \Rightarrow (\forall X4. \\ & (m1\_subset\_1 X4 (u1\_struct\_0 X1)) \Rightarrow ((X4 \in X2) \Rightarrow (k4\_vectsp\_1 X0 X1 \\ & X3 X4 \in X2)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned}
& \forall X0.(l2\_struct\_0 X0) \Rightarrow (\forall X1.((\neg v2\_struct\_0 X1) \wedge \\
& (l1\_vectsp\_1 X1 X0)) \Rightarrow (\forall X2.((\neg v2\_struct\_0 X2) \wedge (l1\_vectsp\_1 \\
& X2 X0)) \Rightarrow (\forall X3.((v1\_funct\_1 X3) \wedge ((v1\_funct\_2 X3 (k2\_zfmisc\_1 \\
& (u1\_struct\_0 X1) (u1\_struct\_0 X2)) (u1\_struct\_0 X0)) \wedge (m1\_subset\_1 \\
& X3 (k1\_zfmisc\_1 (k2\_zfmisc\_1 (k2\_zfmisc\_1 (u1\_struct\_0 X1) (u1\_struct\_0 \\
& X2)) (u1\_struct\_0 X0)))))) \Rightarrow (k10\_bilinear X0 X1 X2 X3 = ReplSep ( \\
& \text{toset } (\lambda X4 : \iota.m1\_subset\_1 X4 (u1\_struct\_0 X1))) (\lambda X4 : \\
& \iota.\forall X5.(m1\_subset\_1 X5 (u1\_struct\_0 X2)) \Rightarrow (k2\_binop\_1 \\
& (u1\_struct\_0 X1) (u1\_struct\_0 X2) (u1\_struct\_0 X0) X3 X4 X5 = k4\_struct\_0 \\
& X0)) (\lambda X4 : \iota.X4))))))
\end{aligned} \tag{16}$$

**Theorem 1**

$$\begin{aligned}
& \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v13\_algstr\_0 X0) \wedge ((v3\_rlvect\_1 \\
& X0) \wedge ((v4\_rlvect\_1 X0) \wedge ((v1\_vectsp\_1 X0) \wedge (l6\_algstr\_0 X0)))))) \Rightarrow \\
& (\forall X1.((\neg v2\_struct\_0 X1) \wedge (l1\_vectsp\_1 X1 X0)) \Rightarrow (\forall X2. \\
& ((\neg v2\_struct\_0 X2) \wedge (l1\_vectsp\_1 X2 X0)) \Rightarrow (\forall X3.((v1\_funct\_1 \\
& X3) \wedge ((v1\_funct\_2 X3 (k2\_zfmisc\_1 (u1\_struct\_0 X1) (u1\_struct\_0 \\
& X2)) (u1\_struct\_0 X0)) \wedge ((v2\_bilinear X3 X0 X1 X2) \wedge ((v4\_bilinear \\
& X3 X0 X1 X2) \wedge (m1\_subset\_1 X3 (k1\_zfmisc\_1 (k2\_zfmisc\_1 (k2\_zfmisc\_1 \\
& (u1\_struct\_0 X1) (u1\_struct\_0 X2)) (u1\_struct\_0 X0))))))))) \Rightarrow ( \\
& v1\_vectsp\_4 (k10\_bilinear X0 X1 X2 X3) X0 X1)))
\end{aligned}$$