

# t44\_monoid\_1 (TMWfKxXHXAd- dEmxapwS3PKSUvQhfAPQgziA)

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Let  $v1\_xboole\_0 : \iota \Rightarrow o$  be given. Let  $v1\_funct\_1 : \iota \Rightarrow o$  be given. Let  $v1\_funct\_2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k2\_zfmisc\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k1\_zfmisc\_1 : \iota \Rightarrow \iota$  be given. Let  $k2\_binop\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k21\_monoid\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k7\_relset\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_xtuple\_0 : \iota \Rightarrow \iota$  be given. Let  $k4\_tarski : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k2\_xtuple\_0 : \iota \Rightarrow \iota$  be given. Let  $k3\_funct\_2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_funct\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k3\_domain\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k2\_domain\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_binop\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_domain\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v1\_relat\_1 : \iota \Rightarrow o$  be given. Assume the following.

$$\forall X0. \forall X1. (k1\_xtuple\_0 (k4\_tarski X0 X1) = X0) \wedge (k2\_xtuple\_0 (k4\_tarski X0 X1) = X1) \quad (1)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. \forall X3. ((\neg v1\_xboole\_0 X0) \wedge \\ & (((v1\_funct\_1 X2) \wedge ((v1\_funct\_2 X2 X0 X1) \wedge (m1\_subset\_1 X2 (k1\_zfmisc\_1 \\ & (k2\_zfmisc\_1 X0 X1)))))) \wedge (m1\_subset\_1 X3 X0))) \Rightarrow (k3\_funct\_2 X0 \\ & X1 X2 X3 = k1\_funct\_1 X2 X3) \end{aligned} \quad (2)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. ((\neg v1\_xboole\_0 X0) \wedge ((\neg v1\_xboole\_0 \\ & X1) \wedge (m1\_subset\_1 X2 (k2\_zfmisc\_1 X0 X1)))) \Rightarrow (k3\_domain\_1 X0 X1 \\ & X2 = k2\_xtuple\_0 X2) \end{aligned} \quad (3)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. ((\neg v1\_xboole\_0 X0) \wedge ((\neg v1\_xboole\_0 \\ & X1) \wedge (m1\_subset\_1 X2 (k2\_zfmisc\_1 X0 X1)))) \Rightarrow (k2\_domain\_1 X0 X1 \\ & X2 = k1\_xtuple\_0 X2) \end{aligned} \quad (4)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.\forall X3.\forall X4.\forall X5. \\ & ((\neg v1\_xboole\_0 X0)\wedge((\neg v1\_xboole\_0 X1)\wedge(((v1\_funct\_1 X3)\wedge( \\ & v1\_funct\_2 X3 (k2\_zfmisc\_1 X0 X1) X2)\wedge(m1\_subset\_1 X3 (k1\_zfmisc\_1 \\ & (k2\_zfmisc\_1 (k2\_zfmisc\_1 X0 X1) X2))))))\wedge((m1\_subset\_1 X4 X0)\wedge \\ & (m1\_subset\_1 X5 X1))))\Rightarrow(k2\_binop\_1 X0 X1 X2 X3 X4 X5 = k1\_binop\_1 \\ & X3 X4 X5) \end{aligned} \tag{5}$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.\forall X3.((\neg v1\_xboole\_0 X0)\wedge \\ & ((\neg v1\_xboole\_0 X1)\wedge((m1\_subset\_1 X2 X0)\wedge(m1\_subset\_1 X3 X1))))\Rightarrow \\ & (k1\_domain\_1 X0 X1 X2 X3 = k4\_tarSKI X2 X3) \end{aligned} \tag{6}$$

Assume the following.

$$\forall X0.\neg v1\_xboole\_0 (k1\_zfmisc\_1 X0) \tag{7}$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.((\neg v1\_xboole\_0 X0)\wedge(\neg v1\_xboole\_0 X1))\Rightarrow \\ & (\neg v1\_xboole\_0 (k2\_zfmisc\_1 X0 X1)) \end{aligned} \tag{8}$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.\forall X3.((\neg v1\_xboole\_0 X0)\wedge \\ & ((\neg v1\_xboole\_0 X1)\wedge((\neg v1\_xboole\_0 X2)\wedge(((v1\_funct\_1 X3)\wedge((v1\_funct\_2 \\ & X3 (k2\_zfmisc\_1 X0 X1) X2)\wedge(m1\_subset\_1 X3 (k1\_zfmisc\_1 (k2\_zfmisc\_1 \\ & (k2\_zfmisc\_1 X0 X1) X2))))))\wedge((v1\_funct\_1 (k21\_monoid\_1 X0 \\ & X1 X2 X3))\wedge((v1\_funct\_2 (k21\_monoid\_1 X0 X1 X2 X3) (k2\_zfmisc\_1 \\ & (k1\_zfmisc\_1 X0) (k1\_zfmisc\_1 X1)) (k1\_zfmisc\_1 X2))\wedge(m1\_subset\_1 \\ & (k21\_monoid\_1 X0 X1 X2 X3) (k1\_zfmisc\_1 (k2\_zfmisc\_1 (k2\_zfmisc\_1 \\ & (k1\_zfmisc\_1 X0) (k1\_zfmisc\_1 X1)) (k1\_zfmisc\_1 X2)))))) \end{aligned} \tag{9}$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.\forall X3.((\neg v1\_xboole\_0 X0)\wedge \\ & ((\neg v1\_xboole\_0 X1)\wedge((m1\_subset\_1 X2 X0)\wedge(m1\_subset\_1 X3 X1))))\Rightarrow \\ & (m1\_subset\_1 (k1\_domain\_1 X0 X1 X2 X3) (k2\_zfmisc\_1 X0 X1)) \end{aligned} \tag{10}$$

Assume the following.

$$\begin{aligned}
& \forall X0.(\neg v1\_xboole\_0 X0) \Rightarrow (\forall X1.(\neg v1\_xboole\_0 X1) \Rightarrow \\
& \quad (\forall X2.(\neg v1\_xboole\_0 X2) \Rightarrow (\forall X3.((v1\_funct\_1 X3) \wedge \\
& \quad ((v1\_funct\_2 X3 (k2\_zfmisc\_1 X0 X1) X2) \wedge (m1\_subset\_1 X3 (k1\_zfmisc\_1 \\
& \quad (k2\_zfmisc\_1 (k2\_zfmisc\_1 X0 X1) X2)))))) \Rightarrow (\forall X4.((v1\_funct\_1 \\
& \quad X4) \wedge ((v1\_funct\_2 X4 (k2\_zfmisc\_1 (k1\_zfmisc\_1 X0) (k1\_zfmisc\_1 \\
& \quad X1)) (k1\_zfmisc\_1 X2)) \wedge (m1\_subset\_1 X4 (k1\_zfmisc\_1 (k2\_zfmisc\_1 \\
& \quad (k2\_zfmisc\_1 (k1\_zfmisc\_1 X0) (k1\_zfmisc\_1 X1)) (k1\_zfmisc\_1 \\
& \quad X2)))))) \Rightarrow ((X4 = k21\_monoid\_1 X0 X1 X2 X3) \Leftrightarrow (\forall X5.(m1\_subset\_1 \\
& \quad X5 (k2\_zfmisc\_1 (k1\_zfmisc\_1 X0) (k1\_zfmisc\_1 X1)))) \Rightarrow (k3\_funct\_2 \\
& \quad (k2\_zfmisc\_1 (k1\_zfmisc\_1 X0) (k1\_zfmisc\_1 X1)) (k1\_zfmisc\_1 \\
& \quad X2) X4 X5 = k7\_relset\_1 (k2\_zfmisc\_1 X0 X1) X2 X3 (k2\_zfmisc\_1 (k2\_domain\_1 \\
& \quad (k1\_zfmisc\_1 X0) (k1\_zfmisc\_1 X1) X5) (k3\_domain\_1 (k1\_zfmisc\_1 \\
& \quad X0) (k1\_zfmisc\_1 X1) X5)))))))))
\end{aligned} \tag{11}$$

Assume the following.

$$\begin{aligned}
& \forall X0.((v1\_relat\_1 X0) \wedge (v1\_funct\_1 X0)) \Rightarrow (\forall X1.\forall X2. \\
& \quad k1\_binop\_1 X0 X1 X2 = k1\_funct\_1 X0 (k4\_tarski X1 X2))
\end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned}
& \forall X0.\forall X1.\forall X2.(m1\_subset\_1 X2 (k1\_zfmisc\_1 \\
& \quad (k2\_zfmisc\_1 X0 X1))) \Rightarrow (v1\_relat\_1 X2)
\end{aligned} \tag{13}$$

**Theorem 1**

$$\begin{aligned}
& \forall X0.(\neg v1\_xboole\_0 X0) \Rightarrow (\forall X1.(\neg v1\_xboole\_0 X1) \Rightarrow \\
& \quad (\forall X2.(\neg v1\_xboole\_0 X2) \Rightarrow (\forall X3.((v1\_funct\_1 X3) \wedge \\
& \quad ((v1\_funct\_2 X3 (k2\_zfmisc\_1 X0 X1) X2) \wedge (m1\_subset\_1 X3 (k1\_zfmisc\_1 \\
& \quad (k2\_zfmisc\_1 (k2\_zfmisc\_1 X0 X1) X2)))))) \Rightarrow (\forall X4.(m1\_subset\_1 \\
& \quad X4 (k1\_zfmisc\_1 X0)) \Rightarrow (\forall X5.(m1\_subset\_1 X5 (k1\_zfmisc\_1 \\
& \quad X1)) \Rightarrow (k2\_binop\_1 (k1\_zfmisc\_1 X0) (k1\_zfmisc\_1 X1) (k1\_zfmisc\_1 \\
& \quad X2) (k21\_monoid\_1 X0 X1 X2 X3) X4 X5 = k7\_relset\_1 (k2\_zfmisc\_1 X0 \\
& \quad X1) X2 X3 (k2\_zfmisc\_1 X4 X5)))))))))
\end{aligned}$$