

# t46\_lattice2 (TM- Mzbb1vzam1E1fDnmgL9aD2vYC6W7Lx4yH)

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Let  $v2\_struct\_0 : \iota \Rightarrow o$  be given. Let  $v10\_lattices : \iota \Rightarrow o$  be given. Let  $l3\_lattices : \iota \Rightarrow o$  be given. Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $u1\_struct\_0 : \iota \Rightarrow \iota$  be given. Let  $v1\_xboole\_0 : \iota \Rightarrow o$  be given. Let  $k5\_finsub\_1 : \iota \Rightarrow \iota$  be given. Let  $v1\_funct\_1 : \iota \Rightarrow o$  be given. Let  $v1\_funct\_2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k1\_zfmisc\_1 : \iota \Rightarrow \iota$  be given. Let  $k2\_zfmisc\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $r3\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k3\_funct\_2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_xboole\_0 : \iota$  be given. Let  $k3\_lattice2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_lattice2 : \iota \Rightarrow \iota$  be given. Let  $k2\_lattice2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_funct\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $g3\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v3\_lattices : \iota \Rightarrow o$  be given. Let  $u1\_lattices : \iota \Rightarrow \iota$  be given. Let  $v3\_binop\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $u2\_lattices : \iota \Rightarrow \iota$  be given. Let  $l2\_lattices : \iota \Rightarrow o$  be given. Let  $l1\_lattices : \iota \Rightarrow o$  be given. Let  $k7\_setwiseo : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned}
& \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices \\
& X0))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 X0)) \Rightarrow (\forall X2. \\
& (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (\forall X3.(m1\_subset\_1 X3 \\
& (u1\_struct\_0 (k1\_lattice2 X0))) \Rightarrow (\forall X4.(m1\_subset\_1 X4 \\
& (u1\_struct\_0 (k1\_lattice2 X0))) \Rightarrow (((r3\_lattices (k1\_lattice2 \\
& X0) X3 X4) \wedge ((X1 = X3) \wedge (X2 = X4))) \Rightarrow (r3\_lattices X0 X2 X1))))))
\end{aligned} \tag{1}$$

Assume the following.

$$\begin{aligned}
& \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices \\
& X0))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 X0)) \Rightarrow (\forall X2. \\
& (m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow ((r3\_lattices X0 X1 X2) \Rightarrow (\forall X3. \\
& (m1\_subset\_1 X3 (u1\_struct\_0 (k1\_lattice2 X0))) \Rightarrow (\forall X4. \\
& (m1\_subset\_1 X4 (u1\_struct\_0 (k1\_lattice2 X0))) \Rightarrow (((X1 = X3) \wedge \\
& X2 = X4) \Rightarrow (r3\_lattices (k1\_lattice2 X0) X4 X3))))))
\end{aligned} \tag{2}$$

Assume the following.

$$\begin{aligned}
& \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices \\
& X0))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 X0)) \Rightarrow (\forall X2. \\
& (\neg v1\_xboole\_0 X2) \Rightarrow (\forall X3.(m1\_subset\_1 X3 (k5\_finsub\_1 X2)) \Rightarrow \\
& (\forall X4.((v1\_funct\_1 X4) \wedge ((v1\_funct\_2 X4 X2 (u1\_struct\_0 \\
& X0)) \wedge (m1\_subset\_1 X4 (k1\_zfmisc\_1 (k2\_zfmisc\_1 X2 (u1\_struct\_0 \\
& X0)))))) \Rightarrow ((\forall X5.(m1\_subset\_1 X5 X2) \Rightarrow ((X5 \in X3) \Rightarrow (r3\_lattices \\
& X0 (k3\_funct\_2 X2 (u1\_struct\_0 X0) X4 X5) X1))) \Rightarrow ((X3 = k1\_xboole\_0) \vee \\
& (r3\_lattices X0 (k2\_lattice2 X2 X0 X3 X4) X1))))))
\end{aligned} \tag{3}$$

Assume the following.

$$\begin{aligned}
& \forall X0. \forall X1. \forall X2. \forall X3. ((\neg v1\_xboole\_0 X0) \wedge \\
& (((v1\_funct\_1 X2) \wedge ((v1\_funct\_2 X2 X0 X1) \wedge (m1\_subset\_1 X2 (k1\_zfmisc\_1 \\
& (k2\_zfmisc\_1 X0 X1)))))) \wedge (m1\_subset\_1 X3 X0))) \Rightarrow (k3\_funct\_2 X0 \\
& X1 X2 X3 = k1\_funct\_1 X2 X3)
\end{aligned} \tag{4}$$

Assume the following.

$$\begin{aligned}
& \forall X0. \forall X1. \forall X2. (((v1\_funct\_1 X1) \wedge ((v1\_funct\_2 \\
& X1 (k2\_zfmisc\_1 X0 X0) X0) \wedge (m1\_subset\_1 X1 (k1\_zfmisc\_1 (k2\_zfmisc\_1 \\
& (k2\_zfmisc\_1 X0 X0) X0)))))) \wedge ((v1\_funct\_1 X2) \wedge ((v1\_funct\_2 X2 \\
& (k2\_zfmisc\_1 X0 X0) X0) \wedge (m1\_subset\_1 X2 (k1\_zfmisc\_1 (k2\_zfmisc\_1 \\
& (k2\_zfmisc\_1 X0 X0) X0)))))) \Rightarrow (\forall X3. \forall X4. \forall X5. \\
& (g3\_lattices X0 X1 X2 = g3\_lattices X3 X4 X5) \Rightarrow ((X0 = X3) \wedge ((X1 = X4) \wedge \\
& (X2 = X5))))
\end{aligned} \tag{5}$$

Assume the following.

$$\begin{aligned}
& \forall X0. ((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices \\
& X0))) \Rightarrow ((v3\_lattices (k1\_lattice2 X0)) \wedge (v10\_lattices (k1\_lattice2 \\
& X0)))
\end{aligned} \tag{6}$$

Assume the following.

$$\begin{aligned}
& \forall X0. ((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices \\
& X0))) \Rightarrow ((v1\_funct\_1 (u1\_lattices X0)) \wedge ((v1\_funct\_2 (u1\_lattices \\
& X0) (k2\_zfmisc\_1 (u1\_struct\_0 X0) (u1\_struct\_0 X0)) (u1\_struct\_0 \\
& X0)) \wedge (v3\_binop\_1 (u1\_lattices X0) (u1\_struct\_0 X0))))
\end{aligned} \tag{7}$$

Assume the following.

$$\begin{aligned}
& \forall X0. ((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices \\
& X0))) \Rightarrow ((v1\_funct\_1 (u2\_lattices X0)) \wedge ((v1\_funct\_2 (u2\_lattices \\
& X0) (k2\_zfmisc\_1 (u1\_struct\_0 X0) (u1\_struct\_0 X0)) (u1\_struct\_0 \\
& X0)) \wedge (v3\_binop\_1 (u2\_lattices X0) (u1\_struct\_0 X0))))
\end{aligned} \tag{8}$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0) \wedge (l3\_lattices X0)) \Rightarrow ((\neg v2\_struct\_0 (k1\_lattice2 X0)) \wedge (v3\_lattices (k1\_lattice2 X0))) \quad (9)$$

Assume the following.

$$\forall X0.(l2\_lattices X0) \Rightarrow ((v1\_funct\_1 (u2\_lattices X0)) \wedge ((v1\_funct\_2 (u2\_lattices X0) (k2\_zfmisc\_1 (u1\_struct\_0 X0) (u1\_struct\_0 X0)) (u1\_struct\_0 X0)) \wedge (m1\_subset\_1 (u2\_lattices X0) (k1\_zfmisc\_1 (k2\_zfmisc\_1 (k2\_zfmisc\_1 (u1\_struct\_0 X0) (u1\_struct\_0 X0)) (u1\_struct\_0 X0)))))) \quad (10)$$

Assume the following.

$$\forall X0.(l1\_lattices X0) \Rightarrow ((v1\_funct\_1 (u1\_lattices X0)) \wedge ((v1\_funct\_2 (u1\_lattices X0) (k2\_zfmisc\_1 (u1\_struct\_0 X0) (u1\_struct\_0 X0)) (u1\_struct\_0 X0)) \wedge (m1\_subset\_1 (u1\_lattices X0) (k1\_zfmisc\_1 (k2\_zfmisc\_1 (k2\_zfmisc\_1 (u1\_struct\_0 X0) (u1\_struct\_0 X0)) (u1\_struct\_0 X0)))))) \quad (11)$$

Assume the following.

$$\forall X0.(l3\_lattices X0) \Rightarrow ((l1\_lattices X0) \wedge (l2\_lattices X0)) \quad (12)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.\forall X3.((\neg v1\_xboole\_0 X0) \wedge (((\neg v2\_struct\_0 X1) \wedge ((v10\_lattices X1) \wedge (l3\_lattices X1))) \wedge ((m1\_subset\_1 X2 (k5\_finsub\_1 X0)) \wedge ((v1\_funct\_1 X3) \wedge ((v1\_funct\_2 X3 X0 (u1\_struct\_0 X1)) \wedge (m1\_subset\_1 X3 (k1\_zfmisc\_1 (k2\_zfmisc\_1 X0 (u1\_struct\_0 X1)))))))))) \Rightarrow (m1\_subset\_1 (k3\_lattice2 X0 X1 X2 X3) (u1\_struct\_0 X1)) \quad (13)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.\forall X3.((\neg v1\_xboole\_0 X0) \wedge (((v1\_funct\_1 X2) \wedge ((v1\_funct\_2 X2 X0 X1) \wedge (m1\_subset\_1 X2 (k1\_zfmisc\_1 (k2\_zfmisc\_1 X0 X1)))))) \wedge (m1\_subset\_1 X3 X0))) \Rightarrow (m1\_subset\_1 (k3\_funct\_2 X0 X1 X2 X3) X1) \quad (14)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((v1\_funct\_1 X1) \wedge ((v1\_funct\_2 X1 (k2\_zfmisc\_1 X0 X0) X0) \wedge (m1\_subset\_1 X1 (k1\_zfmisc\_1 (k2\_zfmisc\_1 (k2\_zfmisc\_1 X0 X0) X0)))))) \wedge ((v1\_funct\_1 X2) \wedge ((v1\_funct\_2 X2 (k2\_zfmisc\_1 X0 X0) X0) \wedge (m1\_subset\_1 X2 (k1\_zfmisc\_1 (k2\_zfmisc\_1 (k2\_zfmisc\_1 X0 X0) X0)))))) \Rightarrow ((v3\_lattices (g3\_lattices X0 X1 X2)) \wedge (l3\_lattices (g3\_lattices X0 X1 X2))) \quad (15)$$

Assume the following.

$$\begin{aligned}
& \forall X0.(\neg v1\_xboole\_0 X0) \Rightarrow (\forall X1.((\neg v2\_struct\_0 X1) \wedge \\
& ((v10\_lattices X1) \wedge (l3\_lattices X1))) \Rightarrow (\forall X2.(m1\_subset\_1 \\
& X2 (k5\_finsub\_1 X0)) \Rightarrow (\forall X3.((v1\_funct\_1 X3) \wedge ((v1\_funct\_2 \\
& X3 X0 (u1\_struct\_0 X1)) \wedge (m1\_subset\_1 X3 (k1\_zfmisc\_1 (k2\_zfmisc\_1 \\
& X0 (u1\_struct\_0 X1)))))) \Rightarrow (k3\_lattice2 X0 X1 X2 X3 = k7\_setwiseo \\
& X0 (u1\_struct\_0 X1) (u1\_lattices X1) X2 X3)))
\end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned}
& \forall X0.(\neg v1\_xboole\_0 X0) \Rightarrow (\forall X1.((\neg v2\_struct\_0 X1) \wedge \\
& ((v10\_lattices X1) \wedge (l3\_lattices X1))) \Rightarrow (\forall X2.(m1\_subset\_1 \\
& X2 (k5\_finsub\_1 X0)) \Rightarrow (\forall X3.((v1\_funct\_1 X3) \wedge ((v1\_funct\_2 \\
& X3 X0 (u1\_struct\_0 X1)) \wedge (m1\_subset\_1 X3 (k1\_zfmisc\_1 (k2\_zfmisc\_1 \\
& X0 (u1\_struct\_0 X1)))))) \Rightarrow (k2\_lattice2 X0 X1 X2 X3 = k7\_setwiseo \\
& X0 (u1\_struct\_0 X1) (u2\_lattices X1) X2 X3)))
\end{aligned} \tag{17}$$

Assume the following.

$$\forall X0.(l3\_lattices X0) \Rightarrow (k1\_lattice2 X0 = g3\_lattices (u1\_struct\_0 X0) (u1\_lattices X0) (u2\_lattices X0)) \tag{18}$$

Assume the following.

$$\forall X0.(l3\_lattices X0) \Rightarrow ((v3\_lattices X0) \Rightarrow (X0 = g3\_lattices (u1\_struct\_0 X0) (u2\_lattices X0) (u1\_lattices X0))) \tag{19}$$

**Theorem 1**

$$\begin{aligned}
& \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge (l3\_lattices \\
& X0))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 X0)) \Rightarrow (\forall X2. \\
& (\neg v1\_xboole\_0 X2) \Rightarrow (\forall X3.(m1\_subset\_1 X3 (k5\_finsub\_1 X2)) \Rightarrow \\
& (\forall X4.((v1\_funct\_1 X4) \wedge ((v1\_funct\_2 X4 X2 (u1\_struct\_0 \\
& X0)) \wedge (m1\_subset\_1 X4 (k1\_zfmisc\_1 (k2\_zfmisc\_1 X2 (u1\_struct\_0 \\
& X0)))))) \Rightarrow ((\forall X5.(m1\_subset\_1 X5 X2) \Rightarrow ((X5 \in X3) \Rightarrow (r3\_lattices \\
& X0 X1 (k3\_funct\_2 X2 (u1\_struct\_0 X0) X4 X5)))) \Rightarrow ((X3 = k1\_xboole\_0) \vee \\
& (r3\_lattices X0 X1 (k3\_lattice2 X2 X0 X3 X4))))))
\end{aligned}$$