

## t49\_monoid\_1

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Let  $v1\_xboole\_0 : \iota \Rightarrow o$  be given. Let  $v1\_funct\_1 : \iota \Rightarrow o$  be given. Let  $v1\_funct\_2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k2\_zfmisc\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k1\_zfmisc\_1 : \iota \Rightarrow \iota$  be given. Let  $v1\_binop\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k21\_monoid\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k7\_relset\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k2\_binop\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k7\_relat\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k3\_binop\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_binop\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. (\neg v1\_xboole\_0 X2) \Rightarrow (\forall X3. \\ & ((v1\_funct\_1 X3) \wedge ((v1\_funct\_2 X3 (k2\_zfmisc\_1 X2 X2) X2) \wedge (m1\_subset\_1 \\ & X3 (k1\_zfmisc\_1 (k2\_zfmisc\_1 (k2\_zfmisc\_1 X2 X2) X2)))))) \Rightarrow ((v1\_binop\_1 \\ & X3 X2) \Rightarrow (k7\_relset\_1 (k2\_zfmisc\_1 X2 X2) X2 X3 (k2\_zfmisc\_1 X0 X1) = \\ & k7\_relset\_1 (k2\_zfmisc\_1 X2 X2) X2 X3 (k2\_zfmisc\_1 X1 X0)))) \end{aligned} \quad (1)$$

Assume the following.

$$\begin{aligned} & \forall X0. (\neg v1\_xboole\_0 X0) \Rightarrow (\forall X1. (\neg v1\_xboole\_0 X1) \Rightarrow \\ & (\forall X2. (\neg v1\_xboole\_0 X2) \Rightarrow (\forall X3. ((v1\_funct\_1 X3) \wedge \\ & ((v1\_funct\_2 X3 (k2\_zfmisc\_1 X0 X1) X2) \wedge (m1\_subset\_1 X3 (k1\_zfmisc\_1 \\ & (k2\_zfmisc\_1 (k2\_zfmisc\_1 X0 X1) X2)))))) \Rightarrow (\forall X4. (m1\_subset\_1 \\ & X4 (k1\_zfmisc\_1 X0)) \Rightarrow (\forall X5. (m1\_subset\_1 X5 (k1\_zfmisc\_1 \\ & X1)) \Rightarrow (k2\_binop\_1 (k1\_zfmisc\_1 X0) (k1\_zfmisc\_1 X1) (k1\_zfmisc\_1 \\ & X2) (k21\_monoid\_1 X0 X1 X2 X3) X4 X5 = k7\_relset\_1 (k2\_zfmisc\_1 X0 \\ & X1) X2 X3 (k2\_zfmisc\_1 X4 X5)))))) \end{aligned} \quad (2)$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. \forall X2. \forall X3. (m1\_subset\_1 X2 ( \\ & k1\_zfmisc\_1 (k2\_zfmisc\_1 X0 X1))) \Rightarrow (k7\_relset\_1 X0 X1 X2 X3 = k7\_relat\_1 \\ & X2 X3) \end{aligned} \quad (3)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.\forall X3.(((v1\_funct\_1 X1)\wedge \\ & ((v1\_funct\_2 X1 (k2\_zfmisc\_1 X0 X0) X0)\wedge(m1\_subset\_1 X1 (k1\_zfmisc\_1 \\ & (k2\_zfmisc\_1 (k2\_zfmisc\_1 X0 X0) X0)))))\wedge((m1\_subset\_1 X2 X0)\wedge \\ & (m1\_subset\_1 X3 X0)))\Rightarrow(k3\_binop\_1 X0 X1 X2 X3 = k1\_binop\_1 X1 X2 X3) \end{aligned} \quad (4)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.\forall X3.\forall X4.\forall X5. \\ & ((\neg v1\_xboole\_0 X0)\wedge((\neg v1\_xboole\_0 X1)\wedge(((v1\_funct\_1 X3)\wedge(( \\ & v1\_funct\_2 X3 (k2\_zfmisc\_1 X0 X1) X2)\wedge(m1\_subset\_1 X3 (k1\_zfmisc\_1 \\ & (k2\_zfmisc\_1 (k2\_zfmisc\_1 X0 X1) X2)))))\wedge((m1\_subset\_1 X4 X0)\wedge \\ & (m1\_subset\_1 X5 X1))))))\Rightarrow(k2\_binop\_1 X0 X1 X2 X3 X4 X5 = k1\_binop\_1 \\ & X3 X4 X5) \end{aligned} \quad (5)$$

Assume the following.

$$\forall X0.\neg v1\_xboole\_0 (k1\_zfmisc\_1 X0) \quad (6)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.\forall X3.((\neg v1\_xboole\_0 X0)\wedge \\ & ((\neg v1\_xboole\_0 X1)\wedge((\neg v1\_xboole\_0 X2)\wedge((v1\_funct\_1 X3)\wedge((v1\_funct\_2 \\ & X3 (k2\_zfmisc\_1 X0 X1) X2)\wedge(m1\_subset\_1 X3 (k1\_zfmisc\_1 (k2\_zfmisc\_1 \\ & (k2\_zfmisc\_1 X0 X1) X2))))))))\Rightarrow((v1\_funct\_1 (k21\_monoid\_1 X0 \\ & X1 X2 X3))\wedge((v1\_funct\_2 (k21\_monoid\_1 X0 X1 X2 X3) (k2\_zfmisc\_1 \\ & (k1\_zfmisc\_1 X0) (k1\_zfmisc\_1 X1)) (k1\_zfmisc\_1 X2))\wedge(m1\_subset\_1 \\ & (k21\_monoid\_1 X0 X1 X2 X3) (k1\_zfmisc\_1 (k2\_zfmisc\_1 (k2\_zfmisc\_1 \\ & (k1\_zfmisc\_1 X0) (k1\_zfmisc\_1 X1)) (k1\_zfmisc\_1 X2)))))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.((v1\_funct\_1 X1)\wedge((v1\_funct\_2 X1 (k2\_zfmisc\_1 \\ & X0 X0) X0)\wedge(m1\_subset\_1 X1 (k1\_zfmisc\_1 (k2\_zfmisc\_1 (k2\_zfmisc\_1 \\ & X0 X0) X0)))))\Rightarrow((v1\_binop\_1 X1 X0)\Leftrightarrow(\forall X2.(m1\_subset\_1 X2 \\ & X0)\Rightarrow(\forall X3.(m1\_subset\_1 X3 X0)\Rightarrow(k3\_binop\_1 X0 X1 X2 X3 = k3\_binop\_1 \\ & X0 X1 X3 X2)))) \end{aligned} \quad (8)$$

### Theorem 1

$$\begin{aligned} & \forall X0.(\neg v1\_xboole\_0 X0)\Rightarrow(\forall X1.((v1\_funct\_1 X1)\wedge( \\ & (v1\_funct\_2 X1 (k2\_zfmisc\_1 X0 X0) X0)\wedge(m1\_subset\_1 X1 (k1\_zfmisc\_1 \\ & (k2\_zfmisc\_1 (k2\_zfmisc\_1 X0 X0) X0)))))\Rightarrow((v1\_binop\_1 X1 X0)\Rightarrow \\ & (v1\_binop\_1 (k21\_monoid\_1 X0 X0 X0 X1) (k1\_zfmisc\_1 X0)))) \end{aligned}$$