

t50_pboole

(TMMFwb9osHz6Sy9vEwT3cmgndh43aUrpU5)

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Let $v1_relat_1 : \iota \Rightarrow o$ be given. Let $v4_relat_1 : \iota \Rightarrow \iota \Rightarrow o$ be given. Let $v1_funct_1 : \iota \Rightarrow o$ be given. Let $v1_partfun1 : \iota \Rightarrow \iota \Rightarrow o$ be given. Let $r2_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$ be given. Let $k2_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Let $r6_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$ be given. Let $k3_pboole : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Let $k1_pboole : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall X0. \forall X1. ((v1_relat_1 X1) \wedge ((v4_relat_1 X1 X0) \wedge (v1_funct_1 X1) \wedge (v1_partfun1 X1 X0))) \Rightarrow (r2_pboole X0 (k1_pboole X0) X1) \tag{1}$$

Assume the following.

$$\begin{aligned} & \forall X0. \forall X1. ((v1_relat_1 X1) \wedge ((v4_relat_1 X1 X0) \wedge (v1_funct_1 X1) \wedge (v1_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1_relat_1 X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))) \Rightarrow \\ & (\forall X3. ((v1_relat_1 X3) \wedge ((v4_relat_1 X3 X0) \wedge ((v1_funct_1 X3) \wedge (v1_partfun1 X3 X0)))) \Rightarrow (r6_pboole X0 (k3_pboole X0 X1 (k2_pboole X0 X2 X3)) (k2_pboole X0 (k3_pboole X0 X1 X2) (k3_pboole X0 X1 X3)))) \end{aligned} \tag{2}$$

Assume the following.

$$\forall X0. \forall X1. ((v1_relat_1 X1) \wedge ((v4_relat_1 X1 X0) \wedge (v1_funct_1 X1) \wedge (v1_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1_relat_1 X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))) \Rightarrow ((r2_pboole X0 X1 X2) \Rightarrow (r6_pboole X0 (k3_pboole X0 X1 X2) X1))) \tag{3}$$

Assume the following.

$$\forall X0. \forall X1. ((v1_relat_1 X1) \wedge ((v4_relat_1 X1 X0) \wedge (v1_funct_1 X1) \wedge (v1_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1_relat_1 X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))) \Rightarrow (r2_pboole X0 (k3_pboole X0 X1 X2) X1)) \tag{4}$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(((v1_relat_1 X1)\wedge((v4_relat_1 \\ & X1 X0)\wedge((v1_funct_1 X1)\wedge(v1_partfun1 X1 X0))))\wedge((v1_relat_1 \\ & X2)\wedge((v4_relat_1 X2 X0)\wedge((v1_funct_1 X2)\wedge(v1_partfun1 X2 X0))))))\Rightarrow \\ & ((r6_pboole X0 X1 X2)\Leftrightarrow(X1 = X2)) \end{aligned} \quad (5)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(((v1_relat_1 X1)\wedge((v4_relat_1 \\ & X1 X0)\wedge((v1_funct_1 X1)\wedge(v1_partfun1 X1 X0))))\wedge((v1_relat_1 \\ & X2)\wedge((v4_relat_1 X2 X0)\wedge((v1_funct_1 X2)\wedge(v1_partfun1 X2 X0))))))\Rightarrow \\ & ((v1_relat_1 (k3_pboole X0 X1 X2))\wedge((v4_relat_1 (k3_pboole X0 \\ & X1 X2) X0)\wedge((v1_funct_1 (k3_pboole X0 X1 X2))\wedge(v1_partfun1 (k3_pboole \\ & X0 X1 X2) X0)))))) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(((v1_relat_1 X1)\wedge((v4_relat_1 \\ & X1 X0)\wedge((v1_funct_1 X1)\wedge(v1_partfun1 X1 X0))))\wedge((v1_relat_1 \\ & X2)\wedge((v4_relat_1 X2 X0)\wedge((v1_funct_1 X2)\wedge(v1_partfun1 X2 X0))))))\Rightarrow \\ & ((v1_relat_1 (k2_pboole X0 X1 X2))\wedge((v4_relat_1 (k2_pboole X0 \\ & X1 X2) X0)\wedge((v1_funct_1 (k2_pboole X0 X1 X2))\wedge(v1_partfun1 (k2_pboole \\ & X0 X1 X2) X0)))))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} & \forall X0.(v1_relat_1 (k1_pboole X0)\wedge((v4_relat_1 (k1_pboole \\ & X0) X0)\wedge((v1_funct_1 (k1_pboole X0)\wedge(v1_partfun1 (k1_pboole \\ & X0) X0)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(((v1_relat_1 X1)\wedge((v4_relat_1 \\ & X1 X0)\wedge((v1_funct_1 X1)\wedge(v1_partfun1 X1 X0))))\wedge((v1_relat_1 \\ & X2)\wedge((v4_relat_1 X2 X0)\wedge((v1_funct_1 X2)\wedge(v1_partfun1 X2 X0))))))\Rightarrow \\ & (k3_pboole X0 X1 X2 = k3_pboole X0 X2 X1) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & \forall X0.\forall X1.\forall X2.(((v1_relat_1 X1)\wedge((v4_relat_1 \\ & X1 X0)\wedge((v1_funct_1 X1)\wedge(v1_partfun1 X1 X0))))\wedge((v1_relat_1 \\ & X2)\wedge((v4_relat_1 X2 X0)\wedge((v1_funct_1 X2)\wedge(v1_partfun1 X2 X0))))))\Rightarrow \\ & (k2_pboole X0 X1 X2 = k2_pboole X0 X2 X1) \end{aligned} \quad (10)$$

Theorem 1

$$\begin{aligned} & \forall X0. \forall X1. ((v1_relat_1 X1) \wedge ((v4_relat_1 X1 X0) \wedge \\ & (v1_funct_1 X1) \wedge (v1_partfun1 X1 X0))) \Rightarrow (\forall X2. ((v1_relat_1 \\ & X2) \wedge ((v4_relat_1 X2 X0) \wedge ((v1_funct_1 X2) \wedge (v1_partfun1 X2 X0)))) \Rightarrow \\ & (\forall X3. ((v1_relat_1 X3) \wedge ((v4_relat_1 X3 X0) \wedge ((v1_funct_1 \\ & X3) \wedge (v1_partfun1 X3 X0)))) \Rightarrow (((r2_pboole X0 X1 (k2_pboole X0 X2 \\ & X3)) \wedge (r6_pboole X0 (k3_pboole X0 X1 X3) (k1_pboole X0))) \Rightarrow (r2_pboole \\ & X0 X1 X2)))) \end{aligned}$$