

# t52\_sublemma

(TMYB2sF4e8QuTnhUcZF3p8vMhugJfvhTuRF)

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Let  $m1\_qc\_lang1 : \iota \Rightarrow o$  be given. Let  $m2\_subset\_1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k2\_qc\_lang1 : \iota \Rightarrow \iota$  be given. Let  $k3\_qc\_lang1 : \iota \Rightarrow \iota$  be given. Let  $k16\_subst1 : \iota \Rightarrow \iota$  be given. Let  $k38\_subst1 : \iota \Rightarrow \iota$  be given. Let  $m1\_subst1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k7\_sublemma : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v3\_subst1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k10\_xtuple\_0 : \iota \Rightarrow \iota$  be given. Let  $k7\_subst1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k11\_cqc\_lang : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k2\_sublemma : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k35\_subst1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k32\_subst1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k9\_sublemma : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k9\_qc\_lang1 : \iota \Rightarrow \iota$  be given. Let  $k21\_qc\_lang1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k15\_qc\_lang1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k22\_qc\_lang1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k1\_xtuple\_0 : \iota \Rightarrow \iota$  be given. Let  $k4\_tarski : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k2\_xtuple\_0 : \iota \Rightarrow \iota$  be given. Let  $k2\_zfmisc\_1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k18\_subst1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k24\_subst1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k23\_subst1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k22\_subst1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k19\_subst1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $v1\_xboole\_0 : \iota \Rightarrow o$  be given. Let  $k1\_zfmisc\_1 : \iota \Rightarrow \iota$  be given. Let  $k1\_subst1 : \iota \Rightarrow \iota$  be given. Let  $k34\_subst1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k33\_subst1 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k3\_cqc\_lang : \iota \Rightarrow \iota$  be given. Let  $v1\_sublemma : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k2\_qc\_lang3 : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k13\_subst1 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall X0.(m1\_qc\_lang1 X0) \Rightarrow (\forall X1.(m2\_subset\_1 X1 (k2\_qc\_lang1 \\ X0) (k3\_qc\_lang1 X0)) \Rightarrow (\forall X2.(m1\_subset\_1 X2 (k9\_qc\_lang1 \\ X0)) \Rightarrow ((k21\_qc\_lang1 X0 (k15\_qc\_lang1 X0 X1 X2) = X1) \wedge (k22\_qc\_lang1 \\ X0 (k15\_qc\_lang1 X0 X1 X2) = X2)))) \end{aligned} \tag{1}$$

Assume the following.

$$\begin{aligned} \forall X0.\forall X1.(k1\_xtuple\_0 (k4\_tarski X0 X1) = X0) \wedge (k2\_xtuple\_0 \\ (k4\_tarski X0 X1) = X1) \end{aligned} \tag{2}$$

Assume the following.

$$\begin{aligned} \forall X0.(m1\_qc\_lang1\ X0) \Rightarrow (\forall X1.(m1\_subset\_1\ X1\ (k2\_zfmisc\_1 \\ (k16\_subst1\ X0)\ (k3\_qc\_lang1\ X0))) \Rightarrow (\forall X2.(m1\_subst1 \\ X2\ X0\ X1) \Rightarrow ((v3\_subst1\ X1\ X0) \Rightarrow ((k18\_subst1\ X0\ (k24\_subst1 \\ X0\ X1\ X2) = k15\_qc\_lang1\ X0\ (k23\_subst1\ X0\ X1)\ (k18\_subst1\ X0 \\ (k22\_subst1\ X0\ X1))) \wedge (k19\_subst1\ X0\ (k24\_subst1\ X0\ X1\ X2) = \\ X2)))))) \end{aligned} \quad (3)$$

Assume the following.

$$\begin{aligned} \forall X0.\forall X1.((\neg v1\_xboole\_0\ X0) \wedge ((\neg v1\_xboole\_0\ X1) \wedge \\ (m1\_subset\_1\ X1\ (k1\_zfmisc\_1\ X0)))) \Rightarrow (\forall X2.(m2\_subset\_1 \\ X2\ X0\ X1) \Leftrightarrow (m1\_subset\_1\ X2\ X1)) \end{aligned} \quad (4)$$

Assume the following.

$$\begin{aligned} \forall X0.\forall X1.\forall X2.((m1\_qc\_lang1\ X0) \wedge ((m1\_subset\_1 \\ X1\ (k38\_subst1\ X0)) \wedge (m1\_subset\_1\ X2\ (k3\_qc\_lang1\ X0)))) \Rightarrow (k7\_sublemma \\ X0\ X1\ X2 = k4\_tarski\ X1\ X2) \end{aligned} \quad (5)$$

Assume the following.

$$\begin{aligned} \forall X0.\forall X1.((m1\_qc\_lang1\ X0) \wedge (m1\_subset\_1\ X1\ (k2\_zfmisc\_1 \\ (k9\_qc\_lang1\ X0)\ (k1\_subst1\ X0)))) \Rightarrow (k34\_subst1\ X0\ X1 = k2\_xtuple\_0 \\ X1) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} \forall X0.\forall X1.((m1\_qc\_lang1\ X0) \wedge (m1\_subset\_1\ X1\ (k2\_zfmisc\_1 \\ (k9\_qc\_lang1\ X0)\ (k1\_subst1\ X0)))) \Rightarrow (k33\_subst1\ X0\ X1 = k1\_xtuple\_0 \\ X1) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} \forall X0.\forall X1.((m1\_qc\_lang1\ X0) \wedge (m1\_subset\_1\ X1\ (k38\_subst1 \\ X0))) \Rightarrow (k2\_sublemma\ X0\ X1 = k1\_xtuple\_0\ X1) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} \forall X0.\forall X1.((m1\_qc\_lang1\ X0) \wedge (m1\_subset\_1\ X1\ (k2\_zfmisc\_1 \\ (k16\_subst1\ X0)\ (k3\_qc\_lang1\ X0)))) \Rightarrow (k23\_subst1\ X0\ X1 = k2\_xtuple\_0 \\ X1) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} \forall X0.\forall X1.((m1\_qc\_lang1\ X0) \wedge (m1\_subset\_1\ X1\ (k2\_zfmisc\_1 \\ (k16\_subst1\ X0)\ (k3\_qc\_lang1\ X0)))) \Rightarrow (k22\_subst1\ X0\ X1 = k1\_xtuple\_0 \\ X1) \end{aligned} \quad (10)$$

Assume the following.

$$\forall X0.\forall X1.((m1\_qc\_lang1 X0)\wedge(m1\_subset\_1 X1 (k16\_subst1 X0)))\Rightarrow(k19\_subst1 X0 X1 = k2\_xtuple\_0 X1) \quad (11)$$

Assume the following.

$$\forall X0.\forall X1.((m1\_qc\_lang1 X0)\wedge(m1\_subset\_1 X1 (k16\_subst1 X0)))\Rightarrow(k18\_subst1 X0 X1 = k1\_xtuple\_0 X1) \quad (12)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.((m1\_qc\_lang1 X0)\wedge((m1\_subset\_1 X1 (k3\_qc\_lang1 X0))\wedge(m1\_subset\_1 X2 (k3\_cqc\_lang X0))))\Rightarrow(k11\_cqc\_lang X0 X1 X2 = k15\_qc\_lang1 X0 X1 X2) \quad (13)$$

Assume the following.

$$\forall X0.\forall X1.k2\_xtuple\_0 (k4\_tarski X0 X1) = X1 \quad (14)$$

Assume the following.

$$\forall X0.(m1\_qc\_lang1 X0)\Rightarrow(\neg v1\_xboole\_0 (k38\_subst1 X0)) \quad (15)$$

Assume the following.

$$\forall X0.(m1\_qc\_lang1 X0)\Rightarrow(\neg v1\_xboole\_0 (k3\_qc\_lang1 X0)) \quad (16)$$

Assume the following.

$$\forall X0.(m1\_qc\_lang1 X0)\Rightarrow(\neg v1\_xboole\_0 (k3\_cqc\_lang X0)) \quad (17)$$

Assume the following.

$$\forall X0.\forall X1.((\neg v1\_xboole\_0 X0)\wedge((\neg v1\_xboole\_0 X1)\wedge(m1\_subset\_1 X1 (k1\_zfmisc\_1 X0))))\Rightarrow(\forall X2.(m2\_subset\_1 X2 X0 X1)\Rightarrow(m1\_subset\_1 X2 X0)) \quad (18)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.((m1\_qc\_lang1 X0)\wedge(((v1\_sublemma X1 X0)\wedge(m1\_subset\_1 X1 (k2\_zfmisc\_1 (k16\_subst1 X0) (k3\_qc\_lang1 X0))))\wedge(m1\_subset1 X2 X0 X1)))\Rightarrow(m2\_subset\_1 (k9\_sublemma X0 X1 X2) (k16\_subst1 X0) (k38\_subst1 X0)) \quad (19)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.((m1\_qc\_lang1 X0)\wedge((m1\_subset\_1 X1 (k38\_subst1 X0))\wedge(m1\_subset\_1 X2 (k3\_qc\_lang1 X0))))\Rightarrow(((v1\_sublemma (k7\_sublemma X0 X1 X2) X0)\wedge(m1\_subset\_1 (k7\_sublemma X0 X1 X2) (k2\_zfmisc\_1 (k16\_subst1 X0) (k3\_qc\_lang1 X0)))) \quad (20)$$

Assume the following.

$$\forall X0.(m1\_qc\_lang1 X0) \Rightarrow (m1\_subset\_1 (k3\_qc\_lang1 X0) (k1\_zfmisc\_1 (k2\_qc\_lang1 X0))) \quad (21)$$

Assume the following.

$$\forall X0.(m1\_qc\_lang1 X0) \Rightarrow (m1\_subset\_1 (k3\_cqc\_lang X0) (k1\_zfmisc\_1 (k9\_qc\_lang1 X0))) \quad (22)$$

Assume the following.

$$\forall X0.(m1\_qc\_lang1 X0) \Rightarrow (m1\_subset\_1 (k38\_subst1 X0) (k1\_zfmisc\_1 (k16\_subst1 X0))) \quad (23)$$

Assume the following.

$$\forall X0.\forall X1.((m1\_qc\_lang1 X0) \wedge (m1\_subset\_1 X1 (k16\_subst1 X0))) \Rightarrow (m1\_subset\_1 (k32\_subst1 X0 X1) (k2\_zfmisc\_1 (k9\_qc\_lang1 X0) (k1\_subst1 X0))) \quad (24)$$

Assume the following.

$$\forall X0.\forall X1.((m1\_qc\_lang1 X0) \wedge (m1\_subset\_1 X1 (k38\_subst1 X0))) \Rightarrow (m2\_subset\_1 (k2\_sublemma X0 X1) (k9\_qc\_lang1 X0) (k3\_cqc\_lang X0)) \quad (25)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.((m1\_qc\_lang1 X0) \wedge ((m1\_subset\_1 X1 (k2\_zfmisc\_1 (k16\_subst1 X0) (k3\_qc\_lang1 X0))) \wedge (m1\_subst1 X2 X0 X1))) \Rightarrow (m1\_subset\_1 (k24\_subst1 X0 X1 X2) (k16\_subst1 X0)) \quad (26)$$

Assume the following.

$$\forall X0.\forall X1.((m1\_qc\_lang1 X0) \wedge (m1\_subset\_1 X1 (k16\_subst1 X0))) \Rightarrow (m1\_subset\_1 (k18\_subst1 X0 X1) (k9\_qc\_lang1 X0)) \quad (27)$$

Assume the following.

$$\forall X0.(m1\_qc\_lang1 X0) \Rightarrow (\forall X1.((v1\_sublemma X1 X0) \wedge (m1\_subset\_1 X1 (k2\_zfmisc\_1 (k16\_subst1 X0) (k3\_qc\_lang1 X0)))) \Rightarrow (\forall X2.(m1\_subst1 X2 X0 X1) \Rightarrow ((v3\_subst1 X1 X0) \Rightarrow (k9\_sublemma X0 X1 X2 = k24\_subst1 X0 X1 X2)))) \quad (28)$$

Assume the following.

$$\begin{aligned}
& \forall X0.(m1\_qc\_lang1\ X0) \Rightarrow (\forall X1.(m1\_subset\_1\ X1\ (k2\_zfmisc\_1 \\
& (k9\_qc\_lang1\ X0)\ (k1\_subst1\ X0))) \Rightarrow (((k21\_qc\_lang1\ X0\ (k33\_subst1 \\
& X0\ X1) \in k10\_xtuple\_0\ (k7\_subst1\ X0\ (k21\_qc\_lang1\ X0\ (k33\_subst1 \\
& X0\ X1))\ (k33\_subst1\ X0\ X1)\ (k34\_subst1\ X0\ X1))) \Rightarrow (k35\_subst1 \\
& X0\ X1 = k2\_qc\_lang3\ X0\ (k13\_subst1\ X0\ (k7\_subst1\ X0\ (k21\_qc\_lang1 \\
& X0\ (k33\_subst1\ X0\ X1))\ (k33\_subst1\ X0\ X1)\ (k34\_subst1\ X0\ X1)) \\
& (k22\_qc\_lang1\ X0\ (k33\_subst1\ X0\ X1)))) \wedge ((\neg k21\_qc\_lang1\ X0 \\
& (k33\_subst1\ X0\ X1) \in k10\_xtuple\_0\ (k7\_subst1\ X0\ (k21\_qc\_lang1 \\
& X0\ (k33\_subst1\ X0\ X1))\ (k33\_subst1\ X0\ X1)\ (k34\_subst1\ X0\ X1))) \Rightarrow \\
& (k35\_subst1\ X0\ X1 = k21\_qc\_lang1\ X0\ (k33\_subst1\ X0\ X1))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\forall X0.(m1\_qc\_lang1\ X0) \Rightarrow (\forall X1.(m1\_subset\_1\ X1\ (k16\_subst1\ X0)) \Rightarrow (k32\_subst1\ X0\ X1 = X1)) \tag{30}$$

Assume the following.

$$\forall X0.(v1\_xboole\_0\ X0) \Rightarrow (\forall X1.(m1\_subset\_1\ X1\ (k1\_zfmisc\_1\ X0)) \Rightarrow (v1\_xboole\_0\ X1)) \tag{31}$$

**Theorem 1**

$$\begin{aligned}
& \forall X0.(m1\_qc\_lang1\ X0) \Rightarrow (\forall X1.(m2\_subset\_1\ X1\ (k2\_qc\_lang1 \\
& X0)\ (k3\_qc\_lang1\ X0)) \Rightarrow (\forall X2.(m2\_subset\_1\ X2\ (k16\_subst1 \\
& X0)\ (k38\_subst1\ X0)) \Rightarrow (\forall X3.(m1\_subst1\ X3\ X0\ (k7\_sublemma \\
& X0\ X2\ X1)) \Rightarrow ((v3\_subst1\ (k7\_sublemma\ X0\ X2\ X1)\ X0) \Rightarrow ((X1 \in k10\_xtuple\_0 \\
& (k7\_subst1\ X0\ X1\ (k11\_cqc\_lang\ X0\ X1\ (k2\_sublemma\ X0\ X2))\ X3)) \vee \\
& (k35\_subst1\ X0\ (k32\_subst1\ X0\ (k9\_sublemma\ X0\ (k7\_sublemma \\
& X0\ X2\ X1)\ X3)) = X1))))))
\end{aligned}$$