

t54\_boolealg  
(TMNB6mo8gmWJHGNpNomJmfDb4BNQmyfe3FG)

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Let  $v2\_struct\_0 : \iota \Rightarrow o$  be given. Let  $v10\_lattices : \iota \Rightarrow o$  be given. Let  $v17\_lattices : \iota \Rightarrow o$  be given. Let  $l3\_lattices : \iota \Rightarrow o$  be given. Let  $m1\_subset\_1 : \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $u1\_struct\_0 : \iota \Rightarrow \iota$  be given. Let  $r1\_boolealg : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k3\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k7\_lattices : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $r4\_boolealg : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow o$  be given. Let  $k4\_lattices : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Let  $k5\_lattices : \iota \Rightarrow \iota$  be given. Let  $v11\_lattices : \iota \Rightarrow o$  be given. Let  $v13\_lattices : \iota \Rightarrow o$  be given. Let  $l1\_lattices : \iota \Rightarrow o$  be given. Let  $l2\_lattices : \iota \Rightarrow o$  be given. Let  $v4\_lattices : \iota \Rightarrow o$  be given. Let  $v15\_lattices : \iota \Rightarrow o$  be given. Let  $v16\_lattices : \iota \Rightarrow o$  be given. Let  $v14\_lattices : \iota \Rightarrow o$  be given. Let  $v5\_lattices : \iota \Rightarrow o$  be given. Let  $v6\_lattices : \iota \Rightarrow o$  be given. Let  $v7\_lattices : \iota \Rightarrow o$  be given. Let  $v8\_lattices : \iota \Rightarrow o$  be given. Let  $v9\_lattices : \iota \Rightarrow o$  be given. Assume the following.

$$\begin{aligned} \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge ((v17\_lattices \\ X0) \wedge (l3\_lattices X0)))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 \\ X0)) \Rightarrow (\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (k7\_lattices \\ X0 (k4\_lattices X0 X1 X2) = k3\_lattices X0 (k7\_lattices X0 X1) (k7\_lattices \\ X0 X2)))) \end{aligned} \tag{1}$$

Assume the following.

$$\begin{aligned} \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge ((v17\_lattices \\ X0) \wedge (l3\_lattices X0)))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 \\ X0)) \Rightarrow (k4\_lattices X0 (k7\_lattices X0 X1) X1 = k5\_lattices X0)) \end{aligned} \tag{2}$$

Assume the following.

$$\begin{aligned} \forall X0.((\neg v2\_struct\_0 X0) \wedge ((v10\_lattices X0) \wedge ((v11\_lattices \\ X0) \wedge (l3\_lattices X0)))) \Rightarrow (\forall X1.(m1\_subset\_1 X1 (u1\_struct\_0 \\ X0)) \Rightarrow (\forall X2.(m1\_subset\_1 X2 (u1\_struct\_0 X0)) \Rightarrow (\forall X3. \\ (m1\_subset\_1 X3 (u1\_struct\_0 X0)) \Rightarrow (k3\_lattices X0 X1 (k4\_lattices \\ X0 X2 X3) = k4\_lattices X0 (k3\_lattices X0 X1 X2) (k3\_lattices X0 X1 \\ X3)))))) \end{aligned} \tag{3}$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge((v10\_lattices X0)\wedge(l3\_lattices X0)))\wedge((m1\_subset\_1 X1 (u1\_struct\_0 X0))\wedge(m1\_subset\_1 X2 (u1\_struct\_0 X0))))\Rightarrow((r1\_boolealg X0 X1 X2)\Leftrightarrow(X1 = X2)) \quad (4)$$

Assume the following.

$$\forall X0.\forall X1.(((\neg v2\_struct\_0 X0)\wedge((v10\_lattices X0)\wedge((v17\_lattices X0)\wedge(l3\_lattices X0))))\wedge(m1\_subset\_1 X1 (u1\_struct\_0 X0)))\Rightarrow(k7\_lattices X0 (k7\_lattices X0 X1) = X1) \quad (5)$$

Assume the following.

$$\forall X0.\forall X1.(((\neg v2\_struct\_0 X0)\wedge((v10\_lattices X0)\wedge((v13\_lattices X0)\wedge(l3\_lattices X0))))\wedge(m1\_subset\_1 X1 (u1\_struct\_0 X0)))\Rightarrow(k3\_lattices X0 (k5\_lattices X0) X1 = X1) \quad (6)$$

Assume the following.

$$\forall X0.(l3\_lattices X0)\Rightarrow((l1\_lattices X0)\wedge(l2\_lattices X0)) \quad (7)$$

Assume the following.

$$\forall X0.\forall X1.(((\neg v2\_struct\_0 X0)\wedge(l3\_lattices X0))\wedge(m1\_subset\_1 X1 (u1\_struct\_0 X0)))\Rightarrow(m1\_subset\_1 (k7\_lattices X0 X1) (u1\_struct\_0 X0)) \quad (8)$$

Assume the following.

$$\forall X0.((\neg v2\_struct\_0 X0)\wedge(l1\_lattices X0))\Rightarrow(m1\_subset\_1 (k5\_lattices X0) (u1\_struct\_0 X0)) \quad (9)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge((v4\_lattices X0)\wedge(l2\_lattices X0)))\wedge((m1\_subset\_1 X1 (u1\_struct\_0 X0))\wedge(m1\_subset\_1 X2 (u1\_struct\_0 X0))))\Rightarrow(m1\_subset\_1 (k3\_lattices X0 X1 X2) (u1\_struct\_0 X0)) \quad (10)$$

Assume the following.

$$\forall X0.\forall X1.\forall X2.(((\neg v2\_struct\_0 X0)\wedge((v4\_lattices X0)\wedge(l2\_lattices X0)))\wedge((m1\_subset\_1 X1 (u1\_struct\_0 X0))\wedge(m1\_subset\_1 X2 (u1\_struct\_0 X0))))\Rightarrow(k3\_lattices X0 X1 X2 = k3\_lattices X0 X2 X1) \quad (11)$$

Assume the following.

$$\begin{aligned} \forall X0. (&l3\_lattices\ X0) \Rightarrow (((\neg v2\_struct\_0\ X0) \wedge (v17\_lattices \\ X0)) \Rightarrow &((\neg v2\_struct\_0\ X0) \wedge ((v11\_lattices\ X0) \wedge ((v15\_lattices \\ X0) \wedge &(v16\_lattices\ X0)))))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} \forall X0. (&l3\_lattices\ X0) \Rightarrow (((\neg v2\_struct\_0\ X0) \wedge (v15\_lattices \\ X0)) \Rightarrow &((\neg v2\_struct\_0\ X0) \wedge ((v13\_lattices\ X0) \wedge (v14\_lattices\ X0)))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} \forall X0. (&l3\_lattices\ X0) \Rightarrow (((\neg v2\_struct\_0\ X0) \wedge (v10\_lattices \\ X0)) \Rightarrow &((\neg v2\_struct\_0\ X0) \wedge ((v4\_lattices\ X0) \wedge ((v5\_lattices\ X0) \wedge \\ ((v6\_lattices\ X0) \wedge &((v7\_lattices\ X0) \wedge ((v8\_lattices\ X0) \wedge (v9\_lattices \\ X0)))))))) \end{aligned} \quad (14)$$

**Theorem 1**

$$\begin{aligned} \forall X0. (&(\neg v2\_struct\_0\ X0) \wedge ((v10\_lattices\ X0) \wedge ((v17\_lattices \\ X0) \wedge (&l3\_lattices\ X0)))) \Rightarrow (\forall X1. (m1\_subset\_1\ X1\ (u1\_struct\_0 \\ X0)) \Rightarrow &(\forall X2. (m1\_subset\_1\ X2\ (u1\_struct\_0\ X0)) \Rightarrow ((r1\_boolealg \\ X0\ (k3\_lattices\ X0\ (k7\_lattices\ X0\ X1)\ (k7\_lattices\ X0\ X2))\ (k3\_lattices \\ X0\ X1\ X2)) \Rightarrow &((r4\_boolealg\ X0\ X1\ (k7\_lattices\ X0\ X1)) \vee ((r4\_boolealg \\ X0\ X2\ (k7\_lattices\ X0\ X2)) \vee ((r1\_boolealg\ X0\ X1\ (k7\_lattices\ X0\ X2)) \wedge \\ (r1\_boolealg\ X0\ X2\ (k7\_lattices\ X0\ X1)))))))) \end{aligned}$$